

CHARACTERISTICS OF COUPLED NONGRAY RADIATING GAS FLOWS

WITH ABLATION PRODUCT EFFECTS ABOUT BLUNT BODIES

DURING PLANETARY ENTRIES

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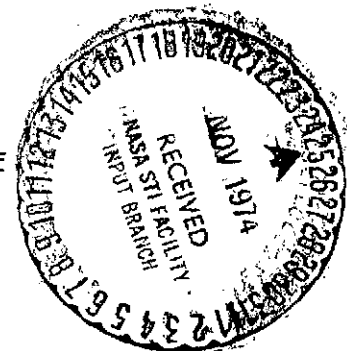
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ABSTRACT

SUTTON, KENNETH. Characteristics of Coupled Nongray Radiating Gas Flows with Ablation Product Effects about Blunt Bodies during Planetary Entries. (Under the direction of FREDERICK OTTO SMETANA).

A computational method is developed for the fully-coupled solution of nongray, radiating gas flows with ablation product effects about blunt bodies during planetary entries. The treatment of radiation accounts for molecular band, continuum, and atomic line transitions with a detailed frequency dependence of the absorption coefficient. The ablation of the entry body is solved as part of the solution for a steady-state ablation process.

Application of the developed method is shown by results at typical conditions for unmanned, scientific probes during entry to Venus. The radiative heating rates along the downstream region of the body can, under certain conditions, exceed the stagnation point value. The radiative heating to the body is attenuated in the boundary layer at the downstream region of the body as well as at the stagnation point of the body.

Results from a study of the radiating, inviscid flow about spherically-capped, conical bodies during planetary entries are presented and show that the nondimensional, radiative heating distributions are nonsimilar with entry conditions. Therefore, extreme caution should be exercised in attempting to extrapolate results from known distributions to other entry conditions for which solutions have not yet been obtained.

BIOGRAPHY

Kenneth Sutton was [REDACTED]. He was reared in Jacksonville, Florida and graduated from Andrew Jackson High School in 1957. He attended Jacksonville University for two years before transferring to the University of Florida where he received the degree of Bachelor of Mechanical Engineering in 1962.

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LIST OF SYMBOLS

| | |
|-------------|--|
| A | parameter defined by eq. 21 |
| a | sonic velocity, see eq. 49 |
| B | parameter defined by eq. 22 |
| B_v | Planck function |
| C | parameter defined by eq. 23 |
| C_1 | parameter defined by eq. A4 |
| C_2 | parameter defined by eq. A5 or eq. 35 |
| C_3 | parameter defined by eq. A6 or eq. 36 |
| c_p | specific heat |
| \bar{c}_p | specific heat parameter defined by eq. 61 |
| D | parameter defined by eq. 24 |
| D_{ij} | binary diffusion coefficient for i th and j th chemical species |
| \bar{D} | self-diffusion coefficient of a reference species |
| E | parameter defined by eq. 25 |
| E_n | exponential integral of order n |
| E_3 | exponential integral of order 3 |
| F^R | divergence of radiative flux defined by eq. 26 |
| F_i | diffusion factor of i th chemical species in a given chemical system, see eq. 57 |
| f | represents the variables p, u, v , or h in eqs. 37 to 39 |
| f | stream function for boundary-layer equations, defined by eq. 70 |
| g_i | metric coefficient for i th coordinate, used herein as $g_1=\lambda$, $g_2=1$, $g_3=r$ (see eqs. 6 to 8) |
| H | total enthalpy, $h+u^2/2$, in boundary-layer equations |

| | |
|-------------------|--|
| h | enthalpy |
| \bar{h} | enthalpy parameter defined by eq. 61 |
| h^* | Planck constant |
| h_a^0 | heat of formation of undecomposed ablation material |
| I_v | monochromatic radiation intensity |
| j_i | diffusional mass flux of chemical element i , see eq. 58 |
| K | body curvature parameter defined by eq. 9 |
| \bar{K}_i | mass fraction of chemical element i |
| k | thermal conductivity |
| ℓ | differential line element in eqs. 6 and 7 |
| M | molecular weight |
| m_a | ablation rate of the heatshield |
| P_1 | thermodynamic variable defined by eqs. 31 and 48 |
| P_3 | thermodynamic variable defined by eqs. 32 and 49 |
| p | pressure |
| q^C | convective heat flux, see eq. 60 |
| q^R | radiative heat flux |
| q_{cond} | heat flux conducted into the heatshield |
| Re_θ | momentum thickness Reynolds number for boundary layer |
| R_n | nose radius of body at $s=0$, see Figure 3 |
| r | radial distance from axis of symmetry of body, see Figure 3 |
| r_b | body radius from axis of symmetry, see Figure 3 |
| S | transformed s coordinate for inviscid-layer equations, defined by eq. 15 |
| SF | stream function for inviscid-layer equations, defined by eq. 45 |

| | |
|------------------|--|
| s | coordinate along the body surface, see Figure 3 |
| T | transformed t coordinate defined by eq. 15 |
| \bar{T} | temperature |
| \bar{T}_δ | temperature at $y=\delta$ in inviscid flow field |
| t | time |
| t_v | dummy variable of monochromatic optical depth in eqs. 76 to 78 |
| u | velocity tangent to body surface, see Figure 3 |
| V_∞ | free-stream velocity |
| V_e | entry velocity |
| v | velocity normal to body surface, see Figure 3 |
| W | weight of entry probe |
| $W/C_d A$ | ballistic coefficient where the drag coefficient, C_d , is for area, A , of the probe's base |
| x_i | general coordinates, used herein as $x_1=s$, $x_2=y$, and $x_3=\phi$ (see eqs. 6 and 7) |
| \bar{x}_j | mole fraction of chemical species j |
| Y | transformed y coordinate, defined by eq. 15 |
| y | coordinate normal to the body surface, see Figure 3 |
| Z_j | quantity for j th chemical species defined by eq. 59 |
| \bar{Z}_i | quantity for i th chemical element defined by eq. 59 |
| z | distance along axis of symmetry of body |
| z | function for the exponential integral of order n , $E_n(z)$ |
| α_{ij} | mass fraction of chemical element i in chemical species j |
| β | difference between the angle of the shock wave and the angle of the body surface, see Figure 3 |
| γ | angle between a ray and the normal for radiation flux in eq. 71 |

| | |
|-------------------|--|
| γ_e | entry angle |
| δ | shock stand-off distance of inviscid layer |
| δ^* | boundary-layer displacement thickness defined by eq. 2 |
| δ^t | total thickness of shock layer defined by eq. 1 |
| ϵ_v | monochromatic emissivity defined by eqs. 76 and 78 |
| ϵ_w | emissivity of the heatshield |
| η | transformed y coordinate for boundary-layer equations, defined by eq. 69 |
| θ | angle of body surface relative to axis of symmetry of body, see Figure 3 |
| θ_c | cone half angle for a spherically-capped, conical body |
| λ | curvature parameter defined by eq. 9 |
| λ_δ | value of λ at $y=\delta$ |
| μ | viscosity in boundary-layer equations |
| μ_1 | parameter defined by eq. 59 |
| μ_2 | parameter defined by eq. 59 |
| μ_v | monochromatic linear absorption coefficient corrected for induced emission |
| $\bar{\mu}_v$ | μ_v for molecular band transitions, defined by eq. 81 |
| ν | radiation frequency |
| ξ | transformed s coordinate for boundary-layer equations, defined by eq. 69 |
| ρ | density |
| $\rho \epsilon_d$ | turbulent eddy diffusivity |
| $\rho \epsilon_h$ | turbulent eddy conductivity |
| $\rho \epsilon_m$ | turbulent eddy viscosity |
| σ | Stefan-Boltzmann constant |

| | |
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| τ_v | monochromatic optical depth defined by eq. 75 |
| τ_w | aerodynamic shear at the wall |
| ϕ | azimuth angle, see Figure 3 |
| Ω | solid angle |

subscripts

| | |
|----------|---|
| a | refers to the ablation material |
| c | refers to the char of the decomposed ablation material |
| e | refers to the boundary-layer edge |
| g | refers to the pyrolysis gas of the decomposed ablation material |
| i,j | refers to the ith or jth chemical species or chemical element |
| o | refers to stagnation-point values, $s=0$ |
| s | refers to conditions behind a normal shock wave |
| w | refers to the body wall |
| ∞ | refers to free-stream conditions |
| δ | refers to conditions at $y=\delta$ in the inviscid layer |
| ν | refers to radiation frequency |

superscripts

| | |
|-------|---|
| C,L | refers to continuum transitions, including molecular band transitions, or line transitions in eq. 79 |
| 1,2 | refers to the first or second step in the two-step numerical method for the radiating, inviscid flow solution |
| +,- | refers to the direction of the radiative flux (+ refers to the radiative flux toward the shock wave and - refers to the radiative flux toward the body) |
| prime | refers to dimensional quantities in the section of Radiating, Inviscid Flow Field Solution |

INTRODUCTION

One of the major scientific endeavors of the present day is the exploration of the planets by unmanned and manned spacecraft. The present technique for planetary entry is to use atmospheric friction as a brake to slow the spacecraft from hypersonic speeds. During hypersonic entry the gas around the body in the formed shock layer is heated by the dissipated kinetic energy. In order for the spacecraft to survive, the main body of the spacecraft will be protected from this source of intense heat by the use of an ablative heat shield material. The speeds required for planetary entries will be well in the excess of the parabolic speed in order to reduce trip times for interplanetary travel. The temperatures of the heated gas behind a shock wave associated with these high-speed entries are sufficient for gas radiation to be a major contributor to the heating of the spacecraft. The inclusion of radiative heating in the energy equation of flow field solutions causes a strong coupling between the radiative transport, the inviscid shock layer, the boundary layer, and the ablative heat shield analyses. Computational methods for the solution of this complex flow phenomena are required before heat shields can be designed to survive during planetary entries at speeds in excess of parabolic speed.

At entry speeds less than parabolic speed the radiative heating is negligible and convection through the boundary layer is the dominant mode of heat transport to the body. The neglect of radiative transport uncouples the inviscid flow equations and the boundary layer equations; however, the boundary layer analysis and the heat shield analysis are

still coupled. Extensive analyses for boundary layers with mass injection have been conducted over the years so that the necessary inputs to the heat shield analysis for convective heating and mass diffusion can be determined without a coupled solution of the equations.

At entry speeds in excess of parabolic speed the radiative transport cannot be neglected and the radiative heating can be much more dominant than convective heating. In general, the radiative transport in a gas is governed by integral equations involving the absorption coefficient integrated over both radiation frequency spectrum and three-dimensional space. The absorption coefficient is dependent on the temperature, density, and the number density of the species present in the gas (ref. 1, page 721). Thus, the radiative transport is dependent on the thermodynamic profiles throughout the gas layer between the shock wave and body. The absorption of radiation by the cooler gas in the boundary layer will change the thermodynamic profiles in the boundary layer and affect the convective heating and mass diffusion to the heat shield. In early studies, the incident radiation to the spacecraft was taken as the radiation from the inviscid shock layer. Later studies have shown that the boundary layer, with and without injection of ablation products, can absorb radiation and reduce the incident radiative heating to the spacecraft. For high-speed planetary entry, the solution of the flow field equations and the heat shield analysis is very complex because of the coupling of radiative transport.

The radiative transport to the stagnation region of blunt bodies for entries into air has been the primary interest for most studies of

radiating flow fields with ablation product effects. The interest for entries into air was due to Earth reentry after exploration of the Earth's moon and preliminary studies for Earth reentry after exploration of Mars. There is a need for investigation of the radiating flow field in other gaseous atmospheres because of the increased interest in exploration of other planets (Venus, Jupiter, etc.). The emphasis on the stagnation region was due to a simplification of the solution and because the radiative heating was considered to be greatest in this region. However, the recent results by Olstad (ref. 2) and by Callis (ref. 3) for an inviscid, radiating flow field have shown that the radiative heating to the flank regions of blunted-conical bodies can equal or exceed the radiative heating to the stagnation region. As discussed by Page (ref. 4), most of the heat shield surface and weight is downstream of the stagnation region and valid treatment of fully-coupled solutions is needed downstream of the stagnation region along cone flanks or over spherical noses. Anderson (ref. 5) states the need for additional solutions of the coupled radiative problem with ablation product effects to complement the existing results. These points illustrate the need for a method of calculating the fully-coupled radiating flow field, with ablation product effects, around the entire body and for entries into atmospheres of arbitrary gases.

The purpose of the present study is to investigate and develop a computational method for a fully-coupled solution of the radiating flow field, with ablation products injection, about blunt bodies for entries into arbitrary gas mixtures. Basically, the method is to separate the

radiating flow field into an outer inviscid layer and an inner boundary layer with injected ablation products; then, the two solutions are coupled by the radiative transport through both layers and by the boundary layer displacement thickness for mass injection. The radiative flux is included in the energy equation of both the inviscid solution and the boundary layer solution. As previously stated, the fully-coupled formulation for radiating planetary entry is very complex and all approaches require varying degrees of approximation in order to obtain a solution. By separating the two different layers of flow, a better treatment should be possible for the solution of each layer.

A second-order, time-asymptotic technique is used for the solution of the inviscid shock layer. The set of unsteady governing equations is hyperbolic, thus the technique is valid in both the subsonic and supersonic regions of the inviscid flow about the body. A computer program for a numerical solution of the technique is developed in the study. An equilibrium thermodynamic subroutine and a detailed radiative transport subroutine are included in the computer program so that arbitrary gas mixtures can be considered. Moretti and Abbett (ref. 6) used a time-asymptotic technique for an inviscid flow solution without radiation in a perfect gas and Callis (refs. 3 and 7) used a time-asymptotic technique for a radiating, inviscid flow solution in air.

The solution of the inner gas layer is for a nonsimilar, multi-component boundary layer with mass injection, unequal diffusion coefficients, and arbitrary chemical species. The numerical, integral matrix method of Barlett and Kendall (ref. 8) is used for the boundary

layer solution and there is an existing computer program called BLIMP. In the computer program, the thermodynamic properties and transport properties (viscosity, etc.) are calculated as part of the solution. The injection rate of ablation products can be either prespecified or calculated during the solution for a steady-state ablation process. The computer program is modified in the present study to account for radiative transport within the boundary layer.

The radiative transport solution for coupling the inviscid layer and the boundary layer is for a nongray gas with molecular band, continuum, and atomic line transitions. The integration over the radiation spectrum uses a detailed frequency dependence of the absorption coefficients and the tangent slab approximation is used for integration over physical space. An existing computer program (RAD/EQUIL) of the numerical method by Nicolet (ref. 9) is used for the solution of the radiative transport. In addition to using this program for coupling the flow fields, an extensive modification of the program is made so that it can be used efficiently as a subroutine in the developed computer program for the inviscid radiating layer.

The application of the present developed method is shown by a study of typical points on an entry trajectory for Venus. To the author's knowledge, there are no published results for a fully-coupled radiating solution with ablation products injection at either the stagnation or downstream regions on a body for Venusian entry. The validity of the method is investigated by a comparison with published results for Earth entry.

REVIEW OF LITERATURE

The review of literature is restricted to prior studies which present the more pertinent results for the radiating flow about blunt bodies with ablation products injection. The fully-coupled solution of radiating flow with ablation analysis is a complex problem and involves many disciplines; therefore, a detailed review of the entire field for the thermal analysis during planetary entry is not presented. A good review and extensive reference list for radiating shock layers is given by Anderson (ref. 5). Similarly, a review of ablative heat shields for planetary entries is given by Walberg and Sullivan (ref. 10). These two articles provide a good review of the present technology of radiative heating and ablation analysis for high-speed planetary entry. For additional discussions of the background information presented in the Introduction on planetary gas dynamics and planetary entry, the reader is referred to references 4 and 11 to 15. Information on trajectory analysis for planetary missions is available in reference 16. A discussion on types of heat shield materials and ablation mechanisms is given in reference 17. A good analysis on the coupling between the boundary layer and a charring ablator is given by Kendall et al., (ref. 18).

The remaining sections of the literature survey present more detailed information which is pertinent to an analysis of the radiating flow about blunt bodies. The first section discusses features of radiation transport modeling. The second section is a discussion on radiating flow field analyses. In addition to flows about blunt bodies, a

discussion is presented on significant and applicable results from stagnation region analyses. Finally, a section is presented on the thermal analyses for Venusian entry.

Radiation Transport Modeling

An atom or molecule in a high-temperature gaseous medium can emit and absorb radiation at a frequency characteristic of some transition from one quantum state to another and each specie has an absorption spectrum which can be expressed in terms of an absorption coefficient. Because of quantum transtions, the absorption coefficient is a function of frequency. Radiation of a given frequency travels in physical space a distance (radiation mean free path) before being absorbed. (The above description is from reference 1, page 721.) Thus, the radiative transport in a gaseous medium is a function of frequency and distance and the equation for the total radiative transport is an integral over both frequency spectrum and physical space.

The solution for a radiating flow field is a set of integro-differential equations and approximations are used to solve this complex system of equations. One of the approximations deals with the spatial aspect of radiative transport. The absorption of radiation at a point in the flow field is a function of the emitted radiation from surrounding fluid elements; and, since radiation travels a distance before being absorbed, the radiative transport is a three-dimensional problem in space. To reduce the complexity of the solution, the "tangent slab approximation"

is usually used for radiating flow fields. This approximation treats the gas layer as a one-dimensional slab in calculation of the radiative transport. The radiative heat flux is neglected except in the direction normal to either the body or shock wave.

Garrett (ref. 19), after a review of the studies of references 20 to 22, states the error introduced by the approximation should be less than 5 percent at the stagnation point. Page (ref. 4), in a discussion of the three-dimensional effects, concludes that the tangent slab approximation is sufficiently accurate for engineering calculations when the shock-layer thickness is small compared to the curvature ($\sim 1/12$). Page's conclusions are based primarily on the studies by Chien (ref. 23) and by Wilson (ref. 24). The study by Wilson was a comparison of the tangent slab approximation with an exact three-dimensional numerical calculation at the stagnation and downstream regions of a blunted, conical body for a typical Earth reentry. The error for the tangent slab approximation was 15 percent at the stagnation region and less error at the conical region. The tangent slab approximation is used in the present study.

In radiation transport modeling, the gas may be treated as transparent, gray, or nongray. A transparent gas is assumed to emit radiation but not absorb radiation. The treatment of a gray or nongray gas allows both emission and absorption. However, the absorption coefficient is assumed constant over the frequency spectrum for a gray gas. A frequency dependence of the absorption coefficient, either approximate or in detail, is included for a nongray gas. Anderson (ref. 5) traces the

historical development of radiating flow fields and shows the progression from transparent, continuum only, uncoupled calculations used by early investigators to the nongray, continuum and line, coupled energy calculations which are current today. As discussed by Anderson, a gray gas analysis is not sufficiently accurate for reentry applications and the gas should be treated as nongray.

The frequency dependence of absorption coefficient for a nongray gas is treated in detail or by a "step-model". In a step model, the frequency dependence is broken into a number of discrete steps (see, for example, references 7 and 25 to 27). The number of steps are generally from two to nine. The calculation of the radiative heat flux involves an integral over frequency and using a step-model will simplify the integration. However, the injection of ablation products into the flow field increases the number of species to be considered and there is a rapid change in chemical composition in the boundary layer. A step-model has to be constructed for the particular gas composition and the addition of ablation products will complicate the construction. Boughner (ref. 25) had difficulty in developing a general step-model for a $\text{CO}_2\text{-N}_2$ mixture; and, the developed step-model was only valid over a restricted and narrow range of conditions. The difficulty was due to molecular band radiation.

The radiative heat flux with ablation product effects has been calculated for a nongray gas and the frequency dependence of absorption coefficient was considered in detail (see, for example, references 19, 28, and 29). There are computer programs for calculating the detailed,

nongray radiative transport of which the ones used primarily for reentry application are RATRAP, developed by Wilson (ref. 30); SPECS, developed by Thomas (ref. 31); and RAD/EQUIL, developed by Nicolet (refs. 9 and 32). Suttles (ref. 33) made a comparison of these programs in a recent study and concluded that the RAD/EQUIL program depended on fewer approximations, included more detail, and required less computer time than RATRAP and SPECS. The RAD/EQUIL program is used in the present study.

As discussed by Penner and Olfe (ref. 13), radiation is coupled to the conservation equations by the radiation pressure, radiation energy density, and radiation energy transfer. The radiation pressure and radiation energy density are usually negligible and can be neglected even when the radiation energy transfer is very important. The two terms can be and are neglected in solutions which deal primarily with radiative heating (refs. 13 and 34). Therefore, the coupling between radiation and the conservation equations is due to the divergence of the radiative heat flux in the energy equation.

For additional information on radiation and radiative transport, the interested reader is referred to the numerous available books. Two good books with which the author is most familiar are by Penner (ref. 35) and by Penner and Olfe (ref. 13). Radiative heating is considered in the book on gas dynamics by Vincenti and Kruger (ref. 34). Also, references 36 to 38 present analyses of radiative transport for high-temperature gases.

Radiating Flow Fields

The interest of the present study is the radiating flow field about blunt bodies; therefore, the flow in both the stagnation region and the downstream region has to be considered. Also, the prior studies which included ablation product effects have been for the stagnation region. A discussion and literature survey are presented first on the pertinent results from analyses of stagnation-region flow; then a discussion and literature survey are presented for flow about blunt bodies.

Flow at Stagnation Region

The recent doctoral thesis by Garrett (ref. 19) presents a detailed review of the various methods used for solving the radiating flow field at the stagnation region. Also, the various methods of solution are discussed in the papers by Goulard et al (ref. 39) and by Anderson (ref. 5). The reader is referred to these papers for a description of the solutions. Only the pertinent results will be presented here.

The earlier studies by Howe and Viegas (ref. 40) and by Hoshizaki and Wilson (refs. 41 and 42) have shown there is a coupling between radiative and convective heating and that radiation from the higher-temperature inviscid region can be absorbed by the lower-temperature viscous region. Later studies have shown that the ablation products injected into the flow field from ablative heat shields can significantly reduce the radiative heating to the body (refs. 19, 22, 28, 29, 43, and 44). These studies were for Earth reentry except a limited part of the study by Wilson (ref. 44) was for Jovian entry. Hoshizaki and

Lasher (ref. 22) state that atomic carbon from the ablation products is the principle absorber and Smith et al. (ref. 28) show that carbon monoxide can also significantly absorb the radiation from the radiating shock layer.

The recent thesis by Garrett (ref. 19) presents a comparison of the results at the stagnation region for air injection and ablation product injection. Garrett uses an implicit finite difference method for a solution at the stagnation-point of a blunt body and treats the entire shock layer as viscous. The results of Garrett show that massive injection of ablation products from a carbon phenolic heat shield into air reduced the radiative heating to the body by 40 percent; but, massive injection of air only reduced the radiative heating by 15 percent. A comparison of results presented by Garrett for solutions from various studies showed that ablation products reduce the radiative heating; but, there were significant differences in the magnitude of the reduction.

Smith et al. (ref. 28) used a method for the stagnation region which is similar to the method in the present study. That is, separating the inviscid layer and the boundary layer and then coupling the two by the radiative transport. The study was for Earth reentry with ablation product effects. The derivation of the boundary layer equations included the radiative heat flux in the energy equation and was for moderate and strong blowing. However, the results presented did not include the radiative heat flux in the energy equation for the boundary layer and was only for strong blowing. The solution of the equations was by numerical methods and Dr. Smith has stated there are difficulties in

extending the solution to include radiative heat flux in the boundary-layer energy equation and to include moderate blowing. The problem has not been solved.* The study by Smith et al. did show that the basic method is valid and in the present study the treatment of the boundary-layer solution is much better than the approximate method used by Smith et al.

Hoshizaki and Lasher (ref. 22) treated the entire shock layer as viscous and stated the advantage is that it eliminates the necessity of matching the radiative heat flux at the inviscid-viscid interface. Garrett (ref. 19) treated the entire layer as viscous and there were irregularities in the enthalpy profiles at the inviscid-viscid interface for strong blowing. The enthalpy profiles come from the energy equation and this equation is where the radiative transport is coupled to the flow equations. In the present study, the coupling is not a direct matching of the radiative heat flux at the interface. The radiative heat flux to be coupled in the energy equation of the boundary layer is taken from the solution of the radiative transport over the entire shock layer. The magnitude of the radiative heat flux from the boundary layer toward the inviscid layer is small compared to the flux toward the boundary layer from the inviscid layer (see, for example, ref. 28). Thus, in the present method the solution for the radiating inviscid layer is solved once and an iteration is only required between the boundary layer solution and radiative transport solution.

*Personal communication with Dr. G. Louis Smith, Langley Research Center, Hampton, Virginia

Garrett (ref. 19) and Rigdon et al. (ref. 29) state the need for properly calculating the thermodynamic properties of the ablation products. In the present study, the thermodynamic properties are calculated during the solution for both the inviscid region and the boundary layer and arbitrary gases can be considered.

As illustrated by the comparisons presented by Garrett (ref. 19), there are discrepancies for the magnitude of radiative heating reduction due to ablation effects even for the stagnation region. The existing results are primarily for Earth reentry and calculations need to be performed for other planetary entries. Anderson (ref. 5) states that realistic calculations for radiative heating to entry vehicles should definitely account for ablation effects and additional radiative coupled analyses with ablation effects should be made in order to complement the existing results.

Flow about Blunt Bodies

There have been a number of studies and numerical solutions for the radiative heating with ablation effects at the blunt-body stagnation region but solutions for the downstream regions of the flow field are not so advanced. The few studies which have considered the radiating flow about blunt bodies have shown that radiative heating at the flank regions of hemisphere-conical bodies can be a large fraction and even exceed the radiative heating at the stagnation point (refs. 2, 3, 45, and 46). These studies were for Earth reentry, used a step-model for the radiative transport, and did not include ablation product effects.

The study by Olstad (ref. 2) was for an inviscid shock layer with the solution of the flow field by the method developed by Maslen (ref. 47) but modified for radiative transport. The method of Maslen is based on the flow being parallel to the shock wave and uses a von Mises transformation. This method uncouples the y-momentum equation. However, the method is not strictly valid at the stagnation region. The study by Olstad was for the Earth reentry of blunted-conical bodies with cone half angles of 30, 45, and 60 degrees. A step-model was used for the radiative transport. The results showed that the radiative heating along the cone flanks was comparable to the stagnation region values for the larger cone angles. Olstad also presented some preliminary results for air injection into the shock layer. These solutions were performed by using the Maslen method for an inner inviscid layer. The results showed that the air injection was more effective in reducing the radiative heating at the stagnation region than at the downstream region. The enthalpy profiles of the inner inviscid layer give the appearance of a boundary layer but are a result of absorption of the radiant energy emitted by the outer inviscid layer.

Callis (ref. 3) investigated the radiating, inviscid flow about long blunt bodies by using a time-asymptotic technique. A discussion of this technique is given below. The study by Callis was for Earth reentry and a step-model was used for the radiative transport. The body shapes were spherically capped cones. The results showed that for larger cone angles (60 degrees half angle) the inviscid radiative heating rates were in excess of the stagnation values over a large portion of the flank region.

Callis states that this increase in heating rates along the cone flanks could pose a serious thermal protection problem.

Chou (ref. 46) presents an approximate method for the flow about blunt bodies in which the entire layer is considered to be viscous. The solution is based on an approximate method for nonsimilar terms, the specie continuity equation is neglected, the density-viscosity product is constant, and the wall conditions are for constant mass injection and temperature along the body. An equation is developed for the nonsimilarity of the tangential velocity and Chou states that the validity of the equation cannot be defined rigorously and the physical reason for the successful application is not clear. The application of the method was for Earth reentry and the injected gas was air. However, radiative transport was not considered for the air injection cases. A comparison was made with the method of Olstad (ref. 2) for inviscid radiating flow. The check cases were supplied by Dr. Olstad and Chou states that his method is valid when the results are compared. However, there is some question about the manner in which the results are being compared and the comparison may not be as good as shown by Chou. The disagreement over the comparison of the results has not been resolved.*

Prior solutions for the radiating flow about blunt bodies did not include ablation product effects and were for Earth reentry. The need for a fully-coupled solution with ablation products for arbitrary gases can be illustrated by the studies of Tauber (refs. 48 and 49) and Tauber

*Personal communications with Dr. W. B. Olstad, Langley Research Center, Hampton, Virginia

and Wakefield (ref. 50). These studies were for planetary entry into Jupiter, Saturn, Uranus, and Neptune with a nominal atmospheric composition of 85 percent hydrogen and 15 percent helium (mole fraction). The basis for the thermal analysis is presented in the paper by Tauber and Wakefield. These studies had to rely on stagnation-point results for the thermal analysis at the downstream regions. Results from the solutions for Earth reentry and the limited results by Wilson (ref. 44) for H_2 -He mixture were used. This recourse had to be used because of the lack of results for flow around bodies with ablation product effects and the lack of results for entry into planets other than Earth. Additional discussion of this problem is given by Page (ref. 4).

There is no method which is generally accepted as being best, even when radiative transport is not included, for the shock layer solution around hypersonic blunt bodies (ref. 39). This point is shown in the paper by Perry and Pasiuk (ref. 51) in which numerous solutions are compared and are compared with experimental data. There is considerable scatter between the different solutions even in the region of subsonic flow. The solution used in the present study for the radiating, inviscid layer is a numerical method of a time-asymptotic technique.

Prior studies have used the time-asymptotic technique for the solution of an inviscid flow field (see, for example, refs. 3, 6, 52 and 53). The time-asymptotic technique retains the time dependent terms in the governing equations of the flow field and permits the time to increase until steady-state conditions are reached. By retaining the time dependent terms, the set of governing equations is hyperbolic and the same

computational method can be used in both the subsonic and supersonic regions of the flow along the body. The development of the technique is due to the studies by Von Neumann and Richtmyer (ref. 54), by Lax (ref. 55), and by Lax and Wendroff (ref. 56). The present method for the solution of the radiating, inviscid flow field is based on the studies of Moretti and Abbett (ref. 6), Callis (ref. 3), and MacCormack (refs. 57 and 58).

Moretti and Abbett (ref. 6) applied the time-asymptotic technique to flow around blunt bodies without radiative transport. The advantages of the method over other methods are discussed briefly by Moretti and Abbett (ref. 6) and more fully by Moretti and Abbett (ref. 59). A comparison of their results with the data presented by Perry and Pasiuk showed the method to be as accurate in matching the experimental data as any other method. Also, it was shown that the shock wave can be treated as a discontinuity and thereby reduce the number of mesh points required in an accurate numerical solution.

Callis (ref. 7), applied the time-asymptotic technique to stagnation-point solutions and Callis (ref. 3), applied the technique for flow about long blunt bodies. The solutions were for radiating, inviscid flow and showed the time-asymptotic technique to be valid for radiating flow fields. Callis stresses the need for treating the shock wave as a discontinuity. In addition to reducing the number of mesh points, the property profiles are more accurately defined for the radiative transport.

In the studies by Callis, and by Moretti and Abbett, a Taylor series expansion with the first three terms retained was used for the advancement of the flow field variables in time. Thus, the solutions are of second-order accuracy in time and require first-order and second-order time derivatives of the flow field variables. A method of second-order accuracy has been developed by MacCormack (refs. 57 and 58) which requires only the first-order time derivatives of the flow field variables. Basically, the method involves a two-step process for the advancement in time with the spatial derivatives of the flow field variables taken in alternate directions. The general method of MacCormack is used in the present study and is described further in the section on Analysis. Anderson (ref. 60) compared the two methods of time advancement in a time-asymptotic technique for the steady-state, nonequilibrium, flow in a nozzle and states the method developed by MacCormack to require fewer computations and 30 percent less computer time for identical results.

The studies by Moretti and Abbett, by Callis, and by MacCormack have shown the numerical methods for the time-asymptotic technique to be convergent, stable, and insensitive to initial guesses. A numerical method of second-order accuracy for the time-asymptotic technique and a discontinuous shock wave is used in the present study for the radiating, inviscid flow field solution.

Thermal Analyses for Venusian Entry

The fully-coupled solution for the radiating flow field with ablation product effects has been investigated in several studies for Earth

reentry (refs. 19, 22, 28, 29, 43, and 44) and Wilson (ref. 44) investigated a solution for Jupiter entry. These solutions were for the stagnation region of the body. At present, there are no published results for a fully-coupled solution with ablation product effects at either the stagnation or downstream regions for Venusian entry. A good review of the present state of technology of thermal analyses and ablation analyses for Venusian entry is given by Walberg and Sullivan (ref. 10).

The primary emphasis at the present time is for unmanned exploration of Venus. A description of the exploration and the scientific experiments is presented in references 61 to 64. Additional information on trajectories, body shapes, and body sizes for unmanned entries is given in references 65 to 69. The entry velocities range from 10 to 15 km/s with the velocities for peak radiative heating being from 7 to 13 km/s. The range of stagnation-point pressure on the body is 1 to 10 atmospheres. The exploration of Venus is a multi-probe, entry mission with one large probe and three small probes. The nominal body for the probes is a spherically capped, conical shape with a .34 meter nose radius and a 60 degree half angle for the large probe and a .14 meter nose radius and a 45 degree half angle for the small probes.

Models of the Venusian atmosphere are given in references 62, 63, and 70. The data for the models are primarily from the flights of Mariner V and Venera 4. Carbon dioxide is the major gas component for the Venusian atmosphere with up to 10 percent by volume of nitrogen.

Reviews of analytical and experimental studies of the radiative heating for Venusian entry and for gas compositions similar to Venusian

atmosphere are presented in references 10, 25, 39, and 71. Therefore, a complete review of the literature will not be given. The results of a recent experimental study in a shock tube for the convective and radiative heating in air and in a .90 CO_2 - .10 N_2 mixture (mole fraction) are presented by Livingston and Williard (ref. 72). A review of the existing studies shows there is a difference in the radiative transport between air and CO_2/N_2 mixtures. The CO_2/N_2 mixtures begin to radiate at a lower temperature than air because of molecular band transitions, primarily CN and CO. At velocities less than 10 km/s the radiative heating can be as much as a factor of 10 greater than air. However, at velocities above 10 km/s the radiative heating is comparable in the two types of gases because of the domination of the atomic processes present.

A detailed study of radiative heating in CO_2/N_2 mixtures is presented in the thesis by Boughner (ref. 25). However, atomic line transitions were neglected and only flight velocities from 8.5 to 10.5 km/s were considered. Most of the study was for a mixture of 50 percent CO_2 and 50 percent N_2 (mole fraction) and the results showed the emission spectrum to be dominated by the $\text{CO}(4+)$ band system and the red and violet bands of CN.

Deacon and Rumpel (ref. 71) present results for the radiative heating of a .80 CO_2 - .20 N_2 mixture (by mole fraction) at flight velocities from 5 to 16.5 km/s. The important radiators were found to be $\text{CO}(4+)$ band system, continuum from carbon, and atomic transitions of neutral carbon. This study was for a constant property layer. Deacon and Rumpel states that to correct for a nonadiabatic layer the corrections

from air studies would have to be applied due to the lack of data for CO_2/N_2 mixtures.

Spiegel et al. (ref. 69), present results for the thermal protection of a spacecraft during a Venusian entry. This investigation was for a completely uncoupled solution. In the study it was assumed: the shock layer is isothermal, no coupling between radiative and convective heating, no radiation absorption by ablation products, and no radiation cooling. The study was for a spherically capped, conical body with a .15 m nose radius and a 60 degree cone half angle. The heat-shield material was high-density phenolic nylon. The results show the radiative heating at a location on the cone flank to be greater by a factor of two than the stagnation-point value. Also, the ablation rates of the heat-shield material are comparable for the cone flank and the stagnation point. These results need to be verified by a fully-coupled solution.

A recent, but unpublished, analysis has been conducted by the present author and R. A. Falanga for the fully-coupled, radiating, flow field solution with ablation products injection at the stagnation point of a blunt body for Venusian entry.* The method of solution was similar to the present method except the radiating, inviscid, flow field solution was by the method of Falanga and Sullivan (ref. 73) and can only be used in the subsonic region of flow about a blunt body. The results of this recent study were that the boundary layer with ablation products injection will reduce the radiative heating to the body by 35 percent at

*Publication pending in Journal of Spacecraft and Rocket.

velocities of 9.86 and 11.18 km/s; but, the reduction was only 14 percent at a velocity of 8.11 km/s. The injection rates were for the steady-state ablation of a high-density, phenolic-nylon, material.

METHOD OF ANALYSIS

The radiating flow field about the entry body is separated into an outer layer where the inviscid flow equations are applicable and an inner viscous layer where the boundary layer equations are applicable. The divergence of the radiative flux is included in the energy equation for the solution of each layer and the boundary layer is coupled to the inviscid shock layer by the radiative transport through both layers and by the boundary layer displacement thickness. This coupling of the boundary layer solution to the inviscid shock layer solution is illustrated by the temperature profile shown in Figure 1. The inviscid flow field is displaced from the wall by the boundary-layer displacement thickness and the boundary-layer profiles are used out to the point where the boundary-layer edge values and their derivatives equal the inviscid layer values. For the boundary-layer solution, the radiative transport is calculated from the mated profiles for the entire layer. The total thickness of the flow field now becomes

$$\delta^t = \delta + \delta^* \quad (1)$$

where δ is the thickness of the inviscid shock layer (shock stand-off distance) and δ^* is the boundary layer displacement thickness calculated from

$$\delta^* = \int_0^{y_e} \left(1 - \frac{\rho u}{\rho_e u_e} \right) dy + \frac{1}{\rho_e u_e r_b} \int_0^s (\rho v)_w r_b ds \quad (2)$$

The first term on the RHS of the expression is the usual term for the displacement thickness of a compressible boundary layer without mass injection and is due to the retardation of the flow in the boundary

layer. The second term is the increase in displacement thickness due to mass injection at the wall for around the body.

The methods of solution of the inviscid flow field, the boundary layer, and the radiative transport are presented in sub-sections of the Method of Analysis. The solution for the radiating, inviscid flow field about a blunt body was developed in the present study and the development of the flow field equations and the numerical method is described in detail. A second-order, time-asymptotic technique is used for the solution. The solution of the inner viscous layer is for a nonsimilar, multicomponent boundary layer with mass injection, unequal diffusion coefficients, and arbitrary chemical species. An existing computer program (BLIMP) is used for the boundary layer solution with modifications made in the present study for coupling the radiative transport and mating the solution to the inviscid flow field. The radiative transport solution is for a nongray gas with molecular band, continuum, and line transitions. An existing computer program (RAD/EQUIL) is used for the solution. In addition to using RAD/EQUIL for coupling the radiative transport through the entire layer, an extensive modification of the program was made in the present study so that it could be used effectively as a subroutine in the developed computer program for the radiative, inviscid flow field.

The radiating, inviscid flow field solution is calculated only once for the fully coupled solution; but, an iteration is required to couple the boundary layer solution to the inviscid flow solution because of inclusion of the radiative heat flux in the boundary layer and locating

the boundary layer edge in the inviscid flow field. The boundary layer solution is assumed not to affect the inviscid layer solution through higher order effects such as recalculating the inviscid solution for an "effective" body composed of the original body plus the displacement thickness. Also, the radiative heat flux from the boundary layer toward the inviscid layer is small compared to the flux from the inviscid layer toward the boundary layer. Thus, the inviscid layer solution should be only slightly affected by the boundary layer solution. The boundary conditions for the boundary layer edge are the appropriate values and their derivatives at the match point in the inviscid flow solution and are not the values at the wall condition of the inviscid flow solution.

The calculation procedure for the fully-coupled solution is shown in Figure 2. The steps in the procedure are:

1. An inviscid, radiating solution is calculated from trajectory data.
2. A boundary layer solution with uncoupled radiative transport is calculated using the inviscid wall values for the boundary layer edge conditions. For steady-state ablation, the radiative heating to the wall is taken as the inviscid wall value.
3. The profiles from the inviscid solution and the boundary layer solution are mated and the radiative transport is calculated for the entire layer.
4. A boundary layer solution with coupled radiative transport is calculated using the values at the match point of the two layers for the boundary layer edge values.
5. Steps 3 and 4 are repeated until the radiative transport through the

layer, the match point condition, and the ablation rates (for steady-state ablation) converge.

The inviscid flow field and the boundary layer solutions are for the flow around a body and the coupling of the two layers and the convergence of the final solution is applied at each location around the body.

The solutions of the radiating inviscid flow, the boundary layer, and the radiative transport for the total layer are calculated by separate computer programs which are coupled through the iteration procedure. Approximately nine hours of total computer time are required for a fully coupled solution about a body and the computer storage requirements for the radiative transport program, the radiating inviscid flow program, and the boundary layer program are 56000_g, 115000_g, and 256000_g, respectively. No attempt has been made to combine the programs into a single computer program because of the large storage requirements and the large computational time.

The effect of the boundary-layer displacement thickness on the inviscid flow solution and subsequent effect on the boundary layer solution could be accounted for by repeating the series of calculations as required until convergence. The computational time for such a procedure would be excessive. Also, the shape of a body is changing during a high velocity entry due to the ablation of the heatshield. Accounting for these shape changes in a solution would require an extensive series of iterations. In the present study, the body shape is the shape prior to entry.

The rate and elemental mass fractions for the mass injection of the ablation products into the boundary layer can be either prespecified or

calculated during a solution for steady-state ablation of a heatshield. The equations for the wall boundary conditions in the boundary layer solution includes the necessary mass and energy balances for steady-state ablation. Steady-state ablation is a good representation of the ablation phenomena at entry conditions where significant radiative heating is present and is better than a specification of the mass injection rate. Steady-state ablation is used for the representation of the ablation of the heatshield for the Venusian entry solutions.

Radiating, Inviscid Flow Field Solution

A time-asymptotic technique is used for the solution of the radiating inviscid flow field; therefore, the analysis is based on the equations of conservation of mass, momentum, and energy for unsteady flow. The conditions for which the present analysis is carried out are that the flow is axisymmetric, inviscid, and nonconducting; the gas is in local thermodynamic and chemical equilibrium; the tangent slab approximation is valid for the radiative transport; and, the shock wave is a discrete surface.

The conservation equations expressed in vector notation are

Mass:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{V}) = 0 \quad (3)$$

Momentum:

$$\frac{\partial \vec{V}}{\partial t} + \vec{V} \cdot \nabla \vec{V} - \frac{1}{\rho} \nabla p = 0 \quad (4)$$

Energy:

$$\rho \frac{\partial h}{\partial t} + \rho \vec{V} \cdot \nabla h + \vec{V} \cdot \vec{q}^R - \frac{\partial p}{\partial t} - \vec{V} \cdot \nabla p = 0 \quad (5)$$

Equations 3 thru 5 are for a general, orthogonal, curvilinear coordinate system. The coordinate system for high speed entries can be either body oriented (direct method) or shock orientated (inverse method). The body-orientated, coordinate system as shown in Figure 3 is used in the present analysis.

Equations 3 thru 5 are transformed to the present, body-orientated, coordinate system by using relationships for vector operations as given in reference 74. The necessary metric coefficients, g_i , for the coordinate transformation are determined from the length of a differential line element and the general expression is

$$(dl)^2 = g_1^2 (dx_1)^2 + g_2^2 (dx_2)^2 + g_3^2 (dx_3)^2 \quad (6)$$

The expression for the present s, y, ϕ coordinate system is

$$(dl)^2 = \lambda^2 (ds)^2 + (dy)^2 + r^2 (d\phi)^2 \quad (7)$$

Comparing this expression to the general expression, the required metric coefficients are

$$\left. \begin{aligned} g_1 &= \lambda \\ g_2 &= 1 \\ g_3 &= r \end{aligned} \right\} \quad (8)$$

where

$$\left. \begin{aligned} r &= y \cos \theta + \int_0^s \sin \theta \, ds \\ \lambda &= 1 + Ky \end{aligned} \right\} \quad (9)$$

$$K = \frac{d\theta}{ds} \quad (9)(\text{concl.})$$

The angle θ is a function of the coordinate s and the magnitude of the angle and the functional relation to s is known from the specification of the body shape.

By knowing the metric coefficients and using the relationships given in reference 74, the vector operations in equations 3 thru 5 can be carried out and the equations determined for the present coordinate system. Applying the axisymmetric conditions that the velocity and all gradients in the ϕ direction are zero, the conservation equations become

Mass:

$$\frac{\partial \rho}{\partial t} + \frac{1}{\lambda} \frac{\partial(\rho u)}{\partial s} + \frac{\rho u}{r} \sin \theta + \frac{\partial(\rho v)}{\partial y} + \rho v \left(\frac{K}{\lambda} + \frac{\cos \theta}{r} \right) = 0 \quad (10)$$

s-momentum:

$$\frac{\partial u}{\partial t} + \frac{u}{\lambda} \frac{\partial u}{\partial s} + v \frac{\partial u}{\partial y} + \frac{Kuv}{\lambda} + \frac{1}{\rho \lambda} \frac{\partial p}{\partial s} = 0 \quad (11)$$

y-momentum:

$$\frac{\partial v}{\partial t} + \frac{u}{\lambda} \frac{\partial v}{\partial s} + v \frac{\partial v}{\partial y} - \frac{Ku^2}{\lambda} + \frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \quad (12)$$

Energy:

$$\frac{\partial h}{\partial t} + \frac{u}{\lambda} \frac{\partial h}{\partial s} + v \frac{\partial h}{\partial y} - \frac{1}{\rho} \frac{\partial p}{\partial t} - \frac{u}{\rho \lambda} \frac{\partial p}{\partial s} - \frac{v}{\rho} \frac{\partial p}{\partial y} + \frac{1}{\rho} \left[\frac{\partial q^R}{\partial y} + q^R \left(\frac{K}{\lambda} + \frac{\cos \theta}{r} \right) \right] = 0 \quad (13)$$

The bracketed term of equation 13 is the divergence of the radiative flux with the application of the tangent slab approximation. In the tangent slab approximation, the derivative of the radiative flux in the y direction is considered in the solution but the derivative in the s

direction is neglected. The parameters in this section of the report are non-dimensionalized by

$$\left. \begin{aligned}
 s &= \frac{s'}{Rn'} & p &= \frac{p'}{\rho_{\infty}' V_{\infty}'^2} & q^R &= \frac{q^R}{\rho_{\infty}' V_{\infty}'^3} \\
 y &= \frac{y'}{Rn'} & h &= \frac{h'}{V_{\infty}'^2} & t &= \frac{t'}{(Rn'/V_{\infty}')} \\
 r &= \frac{r'}{Rn'} & \rho &= \frac{\rho'}{\rho_{\infty}'}
 \end{aligned} \right\} \quad (14)$$

where the subscript ∞ refers to the free stream value and the prime to dimensional quantities.

The value of y at the shock wave will vary with the s location around the body and will also vary with time because the shock wave is moving during the solution until a steady-state solution (final solution) is obtained. It is convenient in the solution to fix the location of the shock wave and this is accomplished by the transformation

$$\left. \begin{aligned}
 T &= t \\
 S &= s \\
 Y &= \frac{y}{\delta}
 \end{aligned} \right\} \quad (15)$$

where

$$\delta = f(S, T)$$

The quantity δ is the shock stand-off distance and the transformation of equations 15 gives $Y=0$ for $y=0$ and $Y=1$ for $y=\delta$. The inviscid flow field is now a rectangular coordinate system of length one for the Y -coordinate and a length for the S -coordinate as specified by the body shape. The value of S is zero at the centerline of the body. The differential

relationships to transform equations 10 thru 13 from the s, y, t coordinate system to the S, Y, T system are

$$\left. \begin{aligned} \frac{\partial(\quad)}{\partial t} &= \frac{\partial(\quad)}{\partial T} - \frac{Y}{\delta} \frac{\partial \delta}{\partial T} \frac{\partial(\quad)}{\partial Y} \\ \frac{\partial(\quad)}{\partial s} &= \frac{\partial(\quad)}{\partial S} - \frac{Y}{\delta} \lambda_{\delta} \tan \beta \frac{\partial(\quad)}{\partial Y} \\ \frac{\partial(\quad)}{\partial y} &= \frac{1}{\delta} \frac{\partial(\quad)}{\partial Y} \end{aligned} \right\} \quad (16)$$

The quantity λ_{δ} is the value of λ evaluated at y equals δ . Hereafter, subscript notation is used to denote partial differentiation with respect to the variables S, Y , and T .

Applying the transformation of equations 16, the conservation equations 10 thru 13 become

Mass:

$$\rho_T = - \left(A \rho_Y + \frac{u}{\lambda} \rho_S - B u_Y + \frac{\rho}{\lambda} u_S + \frac{\rho}{\delta} v_Y + C \right) \quad (17)$$

S-momentum:

$$u_T = - \left(A u_Y + \frac{u}{\lambda} u_S - \frac{B}{\rho} p_Y + \frac{1}{\rho \lambda} p_S + D \right) \quad (18)$$

Y-momentum:

$$v_T = - \left(A v_Y + \frac{u}{\lambda} v_S + \frac{1}{\delta \rho} p_Y - E \right) \quad (19)$$

Energy:

$$h_T = - \left(A h_Y + \frac{u}{\lambda} h_S - \frac{1}{\rho} p_T - \frac{A}{\rho} p_Y - \frac{u}{\rho \lambda} p_S + \frac{F^R}{\rho} \right) \quad (20)$$

where

$$A = \frac{v}{\delta} - \frac{Y}{\delta} \delta_T - \frac{uY}{\delta \lambda} \lambda_{\delta} \tan \beta \quad (21)$$

$$B = \frac{\rho Y}{\delta \lambda} \lambda_{\delta} \tan \beta \quad (22)$$

$$C = \frac{\rho u}{r} \sin \theta + \rho v \left(\frac{K}{\lambda} + \frac{\cos \theta}{r} \right) \quad (23)$$

$$D = \frac{Kuv}{\lambda} \quad (24)$$

$$E = \frac{Ku^2}{\lambda} \quad (25)$$

$$F^R = \frac{1}{\delta} q_Y^R + q^R \left(\frac{K}{\lambda} + \frac{\cos \theta}{r} \right) \quad (26)$$

$$\lambda = 1 + K\delta Y \quad (27)$$

$$\lambda_\delta = 1 + K\delta \quad (28)$$

The mass equation, equation 17, is now changed from density as the dependent variable to pressure as the dependent variable. This is accomplished by using the state equation in the functional form

$$h = h(p, \rho)$$

to obtain the differential relations

$$\left. \begin{aligned} h_T &= \left(\frac{\partial h}{\partial p} \right)_\rho p_T + \left(\frac{\partial h}{\partial \rho} \right)_p \rho_T \\ h_S &= \left(\frac{\partial h}{\partial p} \right)_\rho p_S + \left(\frac{\partial h}{\partial \rho} \right)_p \rho_S \\ h_Y &= \left(\frac{\partial h}{\partial p} \right)_\rho p_Y + \left(\frac{\partial h}{\partial \rho} \right)_p \rho_Y \end{aligned} \right\} \quad (29)$$

Using the differential relations of equations 29 and combining equations 17 and 20, a new equation for the conservation of mass is obtained in the form

Mass:

$$p_T = - \left[A p_Y + \frac{u}{\lambda} p_S + p_3 \left(\frac{\rho}{\lambda} u_S - B u_Y + \frac{\rho}{\delta} v_Y + C \right) - \frac{F^R}{P_1 \rho} \right] \quad (30)$$

where

$$P_1 = \frac{1}{\rho} - \left(\frac{\partial h}{\partial p} \right)_\rho \quad (31)$$

$$P_3 = \frac{1}{P_1} \left(\frac{\partial h}{\partial p} \right)_p \quad (32)$$

The values for the thermodynamic variables of ρ , P_1 , and P_3 are determined by the conditions of chemical equilibrium for the gas with the specification of the elemental mass fractions of the gas and the two thermodynamic state variables of pressure and enthalpy. The dependent variable in the mass equation was changed from density to pressure as the dependent variable because density cannot be used as one of the specified thermodynamic state variables in the chemical equilibrium program used in the present study. Any two of the equations 17, 20, or 30 can be used in the inviscid flow solution depending on the method used for the thermodynamic state.

The basic equations for the solution of the radiating, inviscid flow field are equations 18 thru 28 and equations 30 thru 32. The terms

$$\frac{\rho u \sin \theta}{r}, \frac{\rho v \cos \theta}{r}, \text{ and } \frac{q^R \cos \theta}{r}$$

which appear in equations 23 and 26 are indeterminate at the centerline, $S=0$, and are evaluated by the L'Hospital rule.

The solution for the location of the shock wave and the flow field variables along the shock wave ($Y=1$) is by a method using a quasi, one-dimensional, unsteady characteristic solution and the Rankine-Hugoniot equations for a moving shock. This method is discussed by Moretti and

Abbett (ref. 6) and by Callis (ref. 7). A derivation of the equations for the characteristic solution is presented in Appendix A. The equation for a right running characteristic is

$$\frac{dY}{dT} = A + \frac{\sqrt{P_3}}{\delta} \quad (33)$$

and the compatibility equation is

$$\frac{dv}{dT} + \frac{1}{\rho\sqrt{P_3}} \frac{dp}{dT} = C_2 + \frac{C_3}{\rho\sqrt{P_3}} \quad (34)$$

where

$$C_2 = -\left(\frac{u}{\lambda} v_S - E\right) \quad (35)$$

$$C_3 = -\left[\frac{u}{\lambda} P_S + P_3\left(\frac{\rho}{\lambda} u_S + C\right) - \frac{F^R}{P_1 \rho} - P_3^{Bu_Y}\right] \quad (36)$$

The set of derived equations for the radiating, inviscid flow field about a blunt axisymmetric body is solved by the numerical methods presented below. In addition to the complete body solution, a separate solution for only the stagnation point of a blunt body was obtained in the present study and the equations are presented in Appendix B.

The concept of the time-asymptotic technique is to assume initial values for the flow field variables and then to increment the variables in time until the variables are invariant with time. Thus, the evolution of the flow in time is followed until the flow obtains a steady-state condition. The numerical method used in the present study to obtain a solution is based on the two-step method of MacCormack and a description of the numerical analysis for the method is given in references 57 and 58.

At time T the value of a variable is known at all locations in the flow field. The advancement of a variable from time T to $T+\Delta T$ (one iteration) is given by

$$f(S,Y,T+\Delta T) = f(S,Y,T) + f_T^{\text{ave}} \Delta T \quad (37)$$

where

$$f_T^{\text{ave}} = (f_T^1 + f_T^2)/2 \quad (38)$$

The symbol f represents the variable p , u , v , or h and the superscripts 1 and 2 refer to the first and second step in the two-step process. The time derivative, f_T , of the variable is calculated from the unsteady equations (equations 18, 19, 20, or 30) with finite differences used for the spatial derivatives. The calculation of f_T^{ave} and the advancement from T to $T+\Delta T$ is a two-step process as follows:

1. Compute f_T^1 at time T from the unsteady equations using forward differences for the spatial derivatives. Advance f by

$$f_{n+1} = f_n + f_T^1 \Delta T \quad (39)$$

2. Compute f_T^2 at $n+1$ from the unsteady equations using the variables f_{n+1} and backward differences for the spatial derivatives. Then, compute the average value of the time derivative, f_T^{ave} , by equation 38 and advance the variable, f , to time $T+\Delta T$ by equation 37.

Step 1 is applied to each variable p , u , v and h at all S , Y locations before proceeding to step 2.

A modification to the basic method of MacCormack for the spatial derivatives is used in the present study. In the basic method, two-point, forward finite differences are used for the first step and two-

point, backward differences are used for the second step along all coordinates. In the present method, three-point, central finite differences are used for the derivatives with respect to the S-coordinate for both steps in the two-step process. Three-point, forward differences on the first step and three-point, central differences on the second step are used for the derivatives of h and u with respect to the Y-coordinate. The derivatives of p and v with respect to the Y-coordinate are the basic method of two-point forward on the first step and two-point backward on the second step. A Taylor series expansion is used for the derivation of the finite difference equations for unequal spacing of the nodal points along each coordinate. The same equations are also valid when equally spaced points are specified.

The above numerical method is used for all nodal points except the nodal points along the shock wave ($Y=1$). The flow field variables at the nodal points along the shock wave and the location of the shock wave are calculated by a numerical method using the equations from the quasi, one-dimensional, characteristics analysis and the Rankine-Hugoniot equations for a moving shock wave. The numerical method is illustrated by the diagram in Figure 4. At time T all the quantities are known and the properties at D , δ , and δ_T are desired at time $T+\Delta T$. The solution procedure is as follows:

1. Assume a δ_T at time $T+\Delta T$ and use trapezoidal integration to determine δ by

$$\delta(T+\Delta T) = \delta(T) + \left[\delta_T(T) + \delta_T(T+\Delta T) \right] \frac{\Delta T}{2} \quad (40)$$

The angle β is calculated by

$$\tan \beta(T+\Delta T) = \left(\frac{1}{\lambda_\delta} \frac{\Delta \delta}{\Delta S} \right)_{T+\Delta T} \quad (41)$$

2. The properties of p , u , v , and h at point D are calculated from the Rankine-Hugoniot equations for a moving shock with the known free-stream properties, the assumed δ_T , and calculated β .
3. Point B is located from the characteristics equation (eq. 33) by

$$\Delta Y = \left(A + \frac{\sqrt{P_3}}{\delta} \right)_{\text{ave}} \Delta T \quad (42)$$

where averaged properties of points B and D are used for the integration. The properties at point B are averaged properties of points A and C.

4. A value of v at point D is computed from the integration of the compatibility equation (eq. 34) from point B to point D using averaged properties

$$v_D = v_B + \left(C_2 + \frac{C_3}{\rho \sqrt{P_3}} \right)_{\text{ave}} \Delta T - \left[\left(\frac{1}{\rho \sqrt{P_3}} \right)_{\text{ave}} (P_D - P_B) \right] \quad (43)$$

The value for p_D is the value calculated by the Rankine-Hugoniot equations in step 2.

5. The computed value of v_D from the compatibility equation is compared with the value calculated from the Rankine-Hugoniot equations in step 2. If the values for v_D do not match, then a new value for $\delta_T(T+\Delta T)$ is assumed and steps 1-5 are cycled until convergence.

The numerical method is applied at each time step for each S-coordinate

location beginning with the $S=0$ location. The method is converged at each S location before proceeding to the next location around the body. A Newton-Raphson method is used for successive guesses for δ_T after the first two guesses.

The value of the shock stand-off distance, δ , at each S location is checked by a mass balance. For the mass balance, the mass flow through a plane in the flow field at the particular S location is equal to the mass flow from the free stream through the shock wave up to the particular S location. The expression for δ from the mass balance is

$$\delta = \frac{r_b^2}{2(SF - r_b \cos \theta)} \quad (44)$$

where

$$SF = \int_0^1 (r \rho u) dY \quad (45)$$

To insure a mass balance for the converged solution, the shock stand-off distance at each S location is adjusted if necessary to the value from equation 44 as the solution approaches convergence.

The size of the time step for the advancement at each iteration is calculated by the use of the characteristic equation (eq. 33) in the form

$$\Delta T \leq \frac{\Delta Y}{A + (\sqrt{P_3}/\delta)} \quad (46)$$

This stability criterion is based on the Courant-Friedricks-Lewy condition and insures that the net slope is less than the characteristics slope for hyperbolic equations (ref. 75, chap. 9). Equation 46 is a

one-dimensional equation being used for a two-dimensional problem; however, the ΔY is smaller than ΔS in the problem and no stability problem was encountered. A time step is calculated at each nodal point in the flow field and the minimum value for the nodal points along a ray at a S -coordinate is used for all the points along that particular S -coordinate. There is a large variation in time steps around the body and using a varying time step around the body requires less iterations for convergence than using a minimum time step for the entire flow field. Having a different time plane through the flow field did not affect the converged steady-state solution.

The radiative heat flux, the thermodynamic properties, and the solution of the Rankine-Hugoniot equations are calculated by using the computer program of Nicolet (refs. 9 and 32). A modified version of this program is used as subroutines to the computer program developed for the radiating, inviscid flow field. A discussion of the method of calculating the radiative heat flux and the modifications are presented in the sub-section of Radiative Transport. Large computational times are required for the calculation of the radiative heat flux and the thermodynamic properties; however, these parameters do not have to be updated at each iteration. A study was made and it was determined that the thermodynamic properties and radiative heat flux can be calculated every 50 to 100 iterations without affecting the converged solution but greatly reducing the computational time. For the iterations when the thermodynamic subroutine is not used, the density is calculated by

$$\Delta \rho = \Delta h + \frac{\left(\frac{P_1}{P_3} - \frac{1}{\rho} \right)}{P_1 P_3} \Delta p \quad (47)$$

with the thermodynamic properties P_1 and P_3 held constant. The properties P_1 and P_3 are calculated by

$$P_1 = \frac{\bar{T}c_p}{\rho a^2 \left[\left(\frac{\partial \ln M}{\partial \ln \bar{T}} \right)_p - 1 \right]} \quad (48)$$

$$P_3 = a^2 \quad (49)$$

where \bar{T} is temperature, a is sonic velocity, M is molecular weight, and c_p is specific heat. The non-dimensionalization of the properties is given by

$$\bar{T} = \frac{\bar{T}'}{\bar{T}'_s} \quad a = \frac{a'}{V'_\infty} \quad c_p = \frac{c'_p}{(V'_\infty)^2 / \bar{T}'_s} \quad (50)$$

where the subscript s refers to the conditions behind a normal shock. The expressions are derived from equations 31 and 32 by using thermodynamic relations. The necessary properties for calculating P_1 and P_3 by equations 48 and 49 are available from the thermodynamic subroutine. Pressure and enthalpy which are calculated by the unsteady equations are used as the two specified, thermodynamic state variables for the thermodynamic subroutine and the radiative heat flux subroutine. The solution of the Rankine-Hugoniot equations for a moving shock wave is calculated by the usual Rankine-Hugoniot equations with the velocities on either side of the shock made relative to a fixed shock location. As mentioned, a solution for the Rankine-Hugoniot equations is included in the program of Nicolet.

The initial values to begin a solution are calculated by assuming the shock wave is stationary and the shock stand-off distances around

the body are equal to the shock stand-off distance at the centerline ($S=0$). The value at the centerline is calculated by a correlation from reference 76 of $\delta = .78/\rho_s$ where ρ_s is the nondimensional density behind a normal shock and δ is the nondimensional stand-off distance. The flow field variables for the nodal points along the shock wave are calculated from the Rankine-Hugoniot equations. The variables for the wall nodal points are calculated by assuming an isentropic expansion along the body with a Newtonian pressure distribution. Values for intermediate points in the flow field are calculated by linear interpolation of the shock and body values.

The convergence of a solution is determined by checking the change in enthalpy between iterations. The percentage change in enthalpy between two successive iterations must be less than a specified small value and this criterion must be met for a specified number of iterations. The enthalpy at the body, shock, and midpoint along a ray at each S location is checked.

A computer program was developed for the numerical solution of the radiating, inviscid flow field. The program is written in FORTRAN IV language for CDC 6000 series computers. A listing of the program is given in Appendix C.

Boundary Layer Solution

The boundary layer equations are for a nonsimilar, multicomponent boundary layer with mass injection, unequal diffusion coefficients, and arbitrary chemical species. The solution of the equations is by a

numerical, integral matrix method and there is an existing computer program for the solution called BLIMP. Many types of complex, reactive boundary layer problems can be solved by the program and numerous publications are required for a complete description of the program and the concepts and methods used in a solution. A brief discussion is presented of the basic method and of the modifications to the program made in the present study. A list of the significant publications is given after the discussion of the basic method.

The present discussion of the basic method is from reference 8. The boundary layer equations for an s, y coordinate system of a blunt, axisymmetric body (see Figure 3) are

Mass:

$$\frac{1}{r_b} \frac{\partial(\rho u r_b)}{\partial s} + \frac{\partial(\rho v)}{\partial y} = 0 \quad (51)$$

s-momentum:

$$\rho u \frac{\partial u}{\partial s} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left[(\mu + \rho \epsilon_m) \frac{\partial u}{\partial y} \right] - \frac{\partial p}{\partial s} \quad (52)$$

y-momentum:

$$\frac{\partial p}{\partial y} = 0 \quad (53)$$

Energy:

$$\rho u \frac{\partial H}{\partial s} + \rho v \frac{\partial H}{\partial y} = - \frac{\partial}{\partial y} (q^C + q^R) \quad (54)$$

where

$$H = h + \frac{u^2}{2} \quad (55)$$

Species:

$$\rho u \frac{\partial \bar{K}_i}{\partial s} + \rho v \frac{\partial \bar{K}_i}{\partial y} = \frac{\partial}{\partial y} \left(\rho \epsilon_d \frac{\partial \bar{K}_i}{\partial y} - J_i \right) \quad (56)$$

The species' conservation equation is solved in terms of elemental mass fraction, \bar{K}_i , rather than the species' mole fraction by the application of the Shvab-Zeldovich transformation. This transformation eliminates the chemical source term for chemical equilibrium and reduces the number of species' conservation equations from the number of species to the number of elements. For a boundary layer with ablation products injection, the number of species is typically from 20 to 50 whereas the number of elements is only 2 to 6.

The complexity of using unequal diffusion coefficients for a multicomponent mixture is reduced by a bifurcation approximation for the binary diffusion coefficients. The approximation is expressed as

$$D_{ij} \approx \frac{\bar{D}}{F_i F_j} \quad (57)$$

where \bar{D} is the self-diffusion coefficient of a reference specie and F_i, F_j are the diffusion factors for species i and j . The F_i of a specie for a given chemical system can be determined by finding the set of diffusion factors that gives the minimum total system residual error and a method is presented in reference 77. The bifurcation approximation permits the diffusive flux of element i to be expressed explicitly in terms of properties and gradients of element i and of the system as a whole, but not of other elements. Also, the $I(I-1)/2$ binary diffusion coefficients are replaced by I diffusion factors where I is the number of chemical species.

The diffusional mass flux of element i in the species' conservation equation (eq. 56) can be expressed as

$$j_i = - \frac{\rho \bar{D} \mu_2}{\mu_1 M} \left[\frac{\partial \bar{Z}_i}{\partial y} + (\bar{Z}_i - \bar{K}_i) \frac{\partial \ln \mu_2}{\partial y} \right] \quad (58)$$

by use of the bifurcation approximation and the definition of the new quantities

$$\left. \begin{aligned} Z_j &= \frac{M_j \bar{x}_j}{F_j \mu_2} & \bar{Z}_i &= \sum \alpha_{ij} Z_j \\ \mu_1 &= \sum \bar{x}_j F_j & \mu_2 &= \sum \frac{M_j \bar{x}_j}{F_j} \end{aligned} \right\} \quad (59)$$

The subscript j refers to species and the subscript i refers to elements. The quantity α_{ij} is the mass fraction of element i in species j and Z_j is a quantity which for unequal diffusion lies between a mass fraction and a mole fraction for species j .

The convective heat flux term q^C in the energy equation (eq. 54) is the sum of heat flux due to conduction, diffusion, turbulence, and dissipation of kinetic energy. Using the bifurcation approximation for the heat flux due to diffusion, the convective heat flux can be expressed by

$$q^C = - \left\{ (\rho \epsilon_m + \mu) \frac{\partial (u^2/2)}{\partial y} + (k + \rho \epsilon_h c_p) \frac{\partial \bar{T}}{\partial y} + \rho \epsilon_d \left(\frac{\partial h}{\partial y} - c_p \frac{\partial \bar{T}}{\partial y} \right) + \frac{\rho \bar{D} \mu_2}{\mu_1 M} \left[\frac{\partial \bar{h}}{\partial y} - \bar{c}_p \frac{\partial \bar{T}}{\partial y} + (\bar{h} - h) \frac{\partial \ln \mu_2}{\partial y} \right] \right\} \quad (60)$$

where

$$\bar{h} = \sum Z_j h_j \quad \bar{c}_p = \sum Z_j c_{p_j} \quad (61)$$

The turbulent transport terms in the equations are expressed in the Boussinesq form of eddy viscosity, eddy diffusion, and eddy

conductivity. If turbulent transport is considered, the turbulent transport properties are included in the boundary layer equations when a prespecified Reynolds number is exceeded. The Reynolds number is based on edge properties and momentum thickness. A transition region between laminar flow to turbulent flow is not included.

The boundary conditions at the boundary layer edge are the specifications of u , H , and \bar{K}_1 and their derivatives with respect to the y -coordinate. The wall boundary conditions allow for the possibility of solving numerous types of problems and thus specification of different variables. However, all the variations include zero slip at the wall, $u_w = 0$, and involves in some form the assignment of values for mass injection rate, $(\rho v)_w$; wall enthalpy or wall temperature, h_w or \bar{T}_w ; and elemental mass fractions at the wall, $\bar{K}_{1,w}$.

The equations of the wall boundary conditions for energy balance and elemental mass balance allow a solution for steady-state ablation of a heatshield. The energy balance is

$$-q_w^C - q_w^R - \sigma \epsilon_w \bar{T}_w^4 - (\rho v)_w h_w + m_g h_g + m_c h_c - q_{\text{cond}} = 0 \quad (62)$$

where the subscripts g and c refer to the pyrolysis gas and char for ablation and q_{cond} is the heat conducted into the ablation material at the wall surface. For steady-state ablation

$$m_g h_g + m_c h_c - q_{\text{cond}} = m_a h_a^0 \quad (63)$$

where m_a is the total ablation rate, m_g plus m_c , and h_a^0 is the heat of

formation of the undecomposed ablation material. Noting that

$$(\rho v)_w = m_a \quad (64)$$

then the surface energy balance for steady-state ablation is

$$-q_w^C - q_w^R - \sigma \epsilon_w \bar{T}_w^4 + m_a (h_a^O - h_w) = 0 \quad (65)$$

The elemental mass balance is

$$m_g \bar{K}_{i,g} + m_c \bar{K}_{i,c} - (\rho v)_w \bar{K}_{i,w} - j_{i,w} = 0 \quad (66)$$

and, for steady-state ablation

$$m_g \bar{K}_{i,g} + m_c \bar{K}_{i,c} = m_a \bar{K}_{i,a} \quad (67)$$

thus, the elemental mass balance for steady-state ablation is

$$m_a (\bar{K}_{i,a} - \bar{K}_{i,w}) - j_{i,w} = 0 \quad (68)$$

A solution for steady-state ablation does not require knowledge of the in-depth response of the ablation material but only the surface interactions between the boundary layer and the ablation material. From equations 65 and 68, it is shown that the additional properties of the elemental mass fractions and heat of formation of the undecomposed ablation material and the wall emissivity are all that is required for a steady-state ablation solution. The solution of the surface balances gives the basic boundary conditions of injection rate, enthalpy, and elemental mass fractions at the wall.

An ξ, η coordinate system is used for the actual solution of the boundary-layer equations by the transformations of

$$\left. \begin{aligned} \xi &= \int_0^s u_e \rho_e \mu_e r_b^2 ds \\ \eta &= \frac{r_b u_e}{\sqrt{2\xi}} \int_0^y \rho dy \end{aligned} \right\} \quad (69)$$

and a stream function of

$$f - f_w = \int_0^\eta \frac{u}{u_e} d\eta \quad (70)$$

The transformed equations will not be presented.

The numerical procedure for the solution of the boundary layer equations is an integral matrix method. The boundary layer is divided into strips in the normal direction (η coordinate) and the conservation equations are integrated across the strips. Across a strip, the primary dependent variables of velocity, total enthalpy, and elemental mass fractions and their derivatives with respect to η are related by Taylor series expansions with cubic or quadratic fits. Implicit, finite difference expressions are used for the derivatives in the streamwise direction (ξ coordinate). The set of simultaneous equations which includes the Taylor series, the conservation equations, and the boundary conditions are solved by general Newton-Raphson iteration using a matrix inversion technique involving successive matrix reductions.

Many aspects of the BLIMP program have been left out in the above discussion. Further information for the boundary layer method and the

BLIMP computer program is presented in the following references. The solution for the boundary layer equations is given in references 8, 18, 78, and 79. The solution is for an arbitrary, multicomponent gas mixture and the method for the thermodynamic properties for chemical equilibrium is given in reference 80 and the method for the mixture transport properties is given in reference 81. A method for considering a boundary layer for a blunt body solution with entropy layer is presented in reference 82. Information on the turbulent transport model is given in references 79, 82, and 83. User's manuals for the BLIMP program are references 84 to 86. The description of a companion analysis for in-depth response of an ablative heatshield is given in reference 87.

Modifications to the BLIMP program were necessary in the present study to clarify the coupling of radiative heating and to make the boundary layer edge conditions compatible with the inviscid flow field. The radiative heating was included in the formulation of the energy equation and in the coding of the BLIMP program; however, the values for the radiative heating were set to zero in the program and no provision was made to input the values. The term $(q^C + q^R)$ was considered as being only convective heating, since q^R was set to zero, and was used in expressions in the program which are only applicable for convective heating. When the radiative heating, q^R , is included in the term $(q^C + q^R)$ these expressions are invalid. There was a provision for inputting radiative heating to the wall, unattenuated by the boundary layer, for use in the wall energy balance for steady-state ablation. The convective heating for the energy balance was represented by the

term $(q^C + q^R)$ at the wall and if the radiative heating through the layer is included in the term by q^R then the radiative heating at the wall is accounted for twice and the energy balance is invalid. The BLIMP program was modified by re-defining certain expressions to clarify the difference between convective and radiative heating and to properly input the radiative heating at the nodal locations.

The derivatives for the edge boundary conditions are set in the BLIMP program as the usual boundary layer edge condition of

$$\left(\frac{\partial u}{\partial y}\right)_e = 0 \quad \left(\frac{\partial H}{\partial y}\right)_e = 0 \quad \left(\frac{\partial \bar{K}_1}{\partial y}\right)_e = 0$$

These boundary conditions consider that u , H , and \bar{K}_1 are invariant with y in the inviscid layer. The edge conditions of zero for the velocity and total enthalpy gradients may not be of sufficient accuracy and may give incorrect results for the boundary layer solution because of non-adiabatic effects when radiative transport is present and when an entropy layer is present in the inviscid flow field of a blunt body.

There are several methods in BLIMP for considering the effect of an entropy layer for a blunt body and for these methods the derivatives are specified by appropriate values and not set to zero. However, the methods did not consider the effect on the derivatives for a non-adiabatic inviscid flow field from radiative transport and were not appropriate for the present study. The BLIMP program was modified in the present study so that the derivatives of velocity and total enthalpy with respect to the y coordinate can be specified by inputs and correctly transformed to the η coordinate. Also, a modification was made for specifying the total

enthalpy at the boundary layer edge around the body. These modifications allow for equating the boundary layer edge values with the values of the radiating, inviscid flow field at the match point in the flow field.

Radiative Transport Solution

The solution of the radiative transport is for a nongray gas with molecular band, continuum, and atomic line transitions. A detailed frequency dependence of the absorption coefficients is used in the integration over the radiation spectrum and the tangent slab approximation is used for integration over physical space. An existing computer program (RAD/EQUIL) by Nicolet (refs. 9 and 32) is used for the calculation of the radiative transport. The present section presents a brief discussion of the method and the modifications to the computer program when used as subroutines for the radiating, inviscid flow field solution.

The tangent slab approximation treats the radiative transport as a one-dimensional problem in the direction normal to the body. Thus, the equations for the radiative transport are derived for a gas confined between two infinite parallel boundaries and the property gradients of the gas are considered as zero except in the normal direction between the boundaries. For the present problem, the boundaries are the shock and the body and the normal direction is perpendicular to the body. The tangent slab approximation is applied locally at each location around the body. Note, the tangent slab approximation is used only for the radiative transport calculations and not for the gradients of the thermodynamic state and flow properties for the flow field solution.

The radiative heat flux across a surface at a point in the gas layer is

$$dq_v^R = I_v \cos \gamma d\Omega \quad (71)$$

where I_v is the intensity, γ is the angle between a ray and the normal, and Ω is the solid angle. The basic equation for the intensity gradient along a ray for a gas in local thermodynamic equilibrium is

$$\frac{dI_v}{d\ell} = \mu_v (B_v - I_v) \quad (72)$$

where μ_v is the linear absorption coefficient corrected for induced emission and B_v is the Planck function. The solution of equations 71 and 72 at a point y in the gas layer for the directional, radiative heat flux is given by Nicolet in the form

$$\left. \begin{aligned} q_v^{R,+}(y) &= \pi \int_0^{\epsilon_b^+} B_v(\epsilon_v^+) d\epsilon_v^+ \\ q_v^{R,-}(y) &= \pi \int_0^{\epsilon_b^+} B_v(\epsilon_v^-) d\epsilon_v^- \end{aligned} \right\} \quad (73)$$

where heat fluxes into the slab at the boundaries are neglected. The net flux at point y is

$$q^R(y) = \int_0^\infty q^{R,+}(y) dv - \int_0^\infty q^{R,-}(y) dv \quad (74)$$

The plus sign in the superscripts refers to the heat flux in the direction toward the shock and the negative sign for the flux toward the body.

Nicolet solves the flux equation in a Planck function, B_ν , and emissivity, ϵ_ν , coordinate system where

$$\tau_\nu = \int_0^y \mu_\nu dy \quad (75)$$

$$\left. \begin{aligned} \epsilon_\nu^+ &= 1 - 2E_3(\tau_\nu - t_\nu), \quad t_\nu \leq \tau_\nu \\ \epsilon_\nu^- &= 1 - 2E_3(\tau_\nu - t_\nu), \quad t_\nu \geq \tau_\nu \end{aligned} \right\} \quad (76)$$

where t_ν are the dummy values of optical depth and $E_n(z)$ is the exponential integral of order n . For the solution by Nicolet, the exponential integral of third order is approximated by

$$E_3(z) \approx \frac{\exp(-2z)}{2} \quad (77)$$

and the expressions for emissivity become

$$\left. \begin{aligned} \epsilon_\nu^+ &\approx 1 - \exp [2(t_\nu - \tau_\nu)] \\ \epsilon_\nu^- &\approx 1 - \exp [2(\tau_\nu - t_\nu)] \end{aligned} \right\} \quad (78)$$

The spectral absorption coefficient for a gas mixture is in general

$$\mu_\nu = \sum \mu_i^C(\nu) + \sum \mu_j^L(\nu) \quad (79)$$

where the first term is the continuum contribution with a summation over all continuum transitions and the second term is the line contribution with a summation over all line transitions. The absorption coefficients are calculated in the RAD/EQUIL program and the theoretical expressions,

approximations, and experimental data of the various transitions for the species are given in detail in the report of Nicolet (ref. 9). Transitions of the species N, C, H, O, H^- , C^- , N^- , O^- , H_2 , C_2 , N_2 , O_2 , NO, CO, CN, and N_2^+ are considered for the radiative transport.

In the computer program, the radiative heating is separated into heating due to continuum transitions and line transitions. The continuum contribution is calculated by using only the first term of equation 79 for the spectral absorption coefficient and then the line contribution is calculated by

$$q_{\nu}^{R,L} = q_{\nu}^R - q_{\nu}^{R,C} \quad (80)$$

where the total absorption coefficient is used for the q_{ν}^R value. The molecular transitions are treated as an "equivalent" continuum process by a bandless model in which the bands within each band system are smeared by

$$\bar{\mu}_{\nu} = \frac{\int_{\Delta\nu} \mu_{\nu} d\nu}{\Delta\nu} \quad (81)$$

where the frequency increments $\Delta\nu$ are selected so that $\bar{\mu}_{\nu}$ varies smoothly over the frequency spectrum of the band system. The molecular transitions are included in the continuum contribution to the radiative flux.

For the continuum calculations, the frequency spectrum in terms of photon energy ($h\nu$) is divided into 48 frequency nodal points over the range 0 to 15 ev. The frequency nodal points are not equally spaced over the spectrum but are spaced to resolve the frequency variation of the absorption coefficient especially at photoionization edges. The

number of node points, frequency range, and spacing are inputs to the program and the above values were used in the present study.

The line grouping technique is used for the atomic line transitions. In this technique, the line transitions near a specified frequency value are grouped together and the radiative heating is given as that from the line group. However, each line within the group is treated individually and can assume unshifted Lorentz or Doppler line shapes or a combination of the two with the half widths including the Stark, resonance, and Doppler effects. The frequency grid for each line is composed of 13 nodal points and the grid is dependent upon the characteristics of the gas layer as well as the individual line. Twenty line groups with a total of 134 lines are used in the present study. The line groups are not connected over the frequency spectrum, but are spaced according to the frequency locations of the line transitions.

The thermodynamic properties and species concentrations required for the calculation of the spectral absorption coefficients and Planck functions are calculated in the computer program by a chemical equilibrium method from the specification across the spatial grid of the enthalpy (or temperature), pressure, and elemental mass fractions. Only the species as given above are used for the radiative transport calculations; but, additional species can be included in the chemical equilibrium calculation. A method for the solution of the Rankine-Hugoniot equations for shock waves is also included in the program. The methods for the chemical equilibrium solution and the Rankine-Hugoniot solution are given in the report by Nicolet. The calculation of the thermodynamic state is consistent throughout the fully-coupled solution because the

chemical equilibrium method is the same in both the RAD/EQUIL and the BLIMP programs.

Briefly, the solution procedure for the radiative transport is that the chemical equilibrium calculations are made for the specified spatial grid. The spectral absorption coefficients, Planck functions and optical depths are calculated. Then the emissivities are calculated and the transformation to the B_ν , ϵ_ν coordinates is made by knowing B_ν and ϵ_ν as a function of y . The heat flux equations are then integrated. Cubic integration formulas are used for all integrations except for the frequency integration of the continuum flux. These integrals are evaluated by linear formulas because of the discontinuity at photoionization edges.

The RAD/EQUIL computer program is used for the calculation of the radiative transport through the total layer from the coupling of the boundary layer to the inviscid layer. This is step 3 in the calculation procedure. Also, a modified version of the RAD/EQUIL program is used as a subroutine in the computer program for the radiating, inviscid flow field solution. The following comments refer to the modified version. A separation between the subroutines for the radiative transport calculations and the chemical equilibrium calculations was made in order that the chemical equilibrium calculations can be performed without calculating the radiative transport. The chemical equilibrium subroutine was modified by the addition of a routine to calculate the necessary thermodynamic properties for the evaluation of the P_1 and P_3 properties required for the inviscid flow solution. The required addition was

taken from the chemical equilibrium version in the BLIMP computer program. The method for calculating thermodynamic derivatives for the chemical equilibrium version in BLIMP is given in reference 80. Also, the storage requirement was reduced by deleting options which were not required, by reducing the maximum number of spatial nodal points, and by reducing the maximum number of chemical species in the chemical equilibrium solution. The modified version of the RAD/EQUIL program was then mated to the computer program for the radiating, inviscid flow field solution.

RESULTS AND DISCUSSION

A computational method has been developed for the fully-coupled solution of nongray, radiating gas flow with ablation product effects about blunt bodies during planetary entries. Application of the developed method is shown by results for Venusian entries. There are no published results for a fully-coupled solution of the radiating flow about ablating, planetary entry bodies; therefore, the validity of the present method is shown by comparing the results from solutions to sub-components of the method with published results of corresponding solutions.

The present method for the solution of an inviscid flow field is compared with existing methods for both non-radiating and radiating gas flows about blunt bodies during Earth reentry. Results from solutions by the present method for the radiating, inviscid flow about blunt bodies are also presented for planetary entries. The present method for a fully-coupled solution with ablation product effects is compared with an existing method for the stagnation point of blunt bodies for Earth reentry. Results from the present method are then presented for the fully-coupled solution about blunt bodies for Venusian entries.

Several comments are given about the presentation of the results. In the present coordinate system, the radiative heating directed toward the shock wave is positive and the radiative heating directed toward the body is negative. However, the radiative heating rates at the wall are given by positive values in the presentation of results. As stated in

the section of Radiative Transport Solution, the radiation contributions by molecular band transitions are included in the continuum contribution to the radiative heating. The total contributions of molecular band transitions and continuum transitions are referred to by continuum processes. The line grouping technique is used for the atomic line transitions and the spectral distributions for the line transitions are given by the contribution of the line group and not by the individual lines.

Non-Radiating, Inviscid Air Solution

The present method for an inviscid flow field has been used to obtain a solution for the nonradiating flow about a spherically capped, conical body during an Earth reentry. The results are presented in Figure 5 and are compared with results by the method of Inouye et al. (ref. 88). The method of reference 88 is considered to be one of the more accurate methods and provides a good means of evaluating the accuracy of the present method for an inviscid flow field solution.

The solution is for the free-stream conditions and body shape given in Figure 5(a). The results for the shock shape at the forward region of the body and the location of the sonic line are presented in Figure 5(b). Figure 5(c) presents the results for the shock shape at the downstream region of the body. Because the thickness of the shock layer is much smaller than the dimensions of the body, a better means of comparing the shock shape is the shock stand-off distance and these

results are presented in Figure 5(d). The results for the shock stand-off distance illustrate the inflection points in the shock shape around the body. The results from the present method are in good agreement with the results by the method of reference 88 for the shock shape, especially considering the inflection points, and for the sonic line location.

The results for the distribution of tangential velocity and pressure along the body are presented in Figures 5(e) and 5(f), respectively. The results from the present method are in good agreement with the results by the method of reference 88 even in the region of the over-expansion of the flow at the sphere-cone junction.

The good agreement between the present results and those by the method of reference 88 indicates the present method provides a good solution for the inviscid flow about a blunt body. Radiating flow is not treated by the method of reference 88.

Radiating, Inviscid Solutions About Blunt Bodies

Results are presented from solutions of the radiating, inviscid gas flow about blunt bodies during planetary entry. The present results for solutions of entries into the Earth's atmosphere are compared with existing methods.

The present method is compared with the method of Callis (ref. 3) for an Earth reentry and the results are presented in Figure 6. The free-stream conditions and body shape are given in Figure 6(a). The

distributions of the wall pressure and the shock stand-off distance are presented in Figures 6(b) and 6(c), respectively. The two methods are in good agreement for these parameters. The distribution of wall, radiative heating rate is presented in Figure 6(d). The results by the two methods are in reasonable agreement at the forward region of the body; however, there is a considerable difference between the methods at the downstream region of the body. The results by Callis are greater by a factor of two than the present results at the downstream region.

An analysis was done to explain the difference between the methods. The thermodynamic profiles of temperature and pressure and the shock stand-off distances from the results by Callis were used as inputs to the radiative transport program, (RAD/EQUIL), used in the present method. The results of this calculation are compared with the present results in Figure 6(e). As shown by the data, the distributions of wall, radiative heating rate is now in good agreement at the downstream region. A similar type of analysis had to be used by Suttles (ref. 89) in comparing his results with Callis for large stand-off distances at the stagnation point of blunt bodies. The radiative heating rates by Callis were greater than those of Suttles for larger stand-off distances.

Results from the present method and the method of Callis are in good agreement for nonradiating flow solutions. The difference between the methods for radiating flow is attributed to the radiation models. The radiation model used by Callis (refs. 3 and 7) was developed by Olstad (refs. 2 and 26) and is based on a "step-model" approximation for the frequency dependence of the absorption coefficient. It is

possible that this radiation model by Olstad treats the radiating gas as being too transparent.* Thus, more radiation is transmitted to the body and this effect would be more noticeable the greater the shock stand-off distance.

The present method was used to calculate the radiative heating rate to the stagnation point of a blunt body for a range of nose radii and the results are compared with the method of Callis (ref. 7) in Figure 7. The same radiation model was used by Callis in references 3 and 7. The results by Callis are greater than the present results and the difference increases with the shock stand-off distance (nose radius).

A solution was obtained for the same conditions as previously given in Figure 6(a), except the nose radius was increased to 2.0 meters. The distribution of radiative heating rate is presented in Figure 8. The results of Callis are 50 percent greater than the present results at the forward region of the body as well as the downstream region.

The stagnation-point heating rates from solutions by the present method and the method of Callis are compared with the methods of references 73 and 89 in Table I for two nose radii. The methods of references 73 and 89 can only be used for the flow in the subsonic region of a blunt body. For the smaller nose radius, the results agree within 15 percent. For the larger nose radius, the present results and those by references 78 and 89 agree within 5 percent; but, the result by Callis is 50 percent greater. The radiation model (RATRAP, ref. 30) used by

*Personal Communications with Dr. W. B. Olstad, Langley Research Center, Hampton, Virginia.

references 73 and 89 treats the frequency dependence of absorption coefficient in detail as does the present radiation model (RAD/EQUIL, ref. 9). As stated in the section Radiation Transport Modeling, Suttles (ref. 32) made a comparison of these radiation models and concluded that the RAD/EQUIL program was the better model. The present method is in agreement with other methods which use a detailed radiation model.

The previously available results for the radiating, inviscid flow about blunt bodies have been provided by the analysis of Callis (ref. 3) and Olstad (ref. 2). The step-model approximation developed by Olstad was used in both methods for the treatment of radiation. Olstad uses an approximate method for the solution of the flow field and the method is inverse, shock shape specified, rather than direct, body shape specified, as the present method. For these reasons, the present method was not compared with the method of Olstad. Since the same radiation model was used by Olstad and Callis, the results of Olstad should not agree with results by the present method.

The difference in the results between the present method and the method of Callis (ref. 3) for the radiative heating rates is attributed to the different radiation models used in the methods. The radiation model used in the present method treats the frequency dependence of the absorption coefficient in detail and is a more accurate model than the step model used by Callis. From the above analysis, it is concluded that the present method provides a good solution for the radiating, inviscid gas flow about blunt bodies.

Callis (ref. 3) and Olstad (ref. 2) state that the nondimensional, radiative heating rate distributions along blunt bodies are nonsimilar

with respect to body angle and nose radius. Suttles (ref. 89) states that the nondimensional, radiative heating rate distribution along the body at the subsonic region of flow for hemispheres to be relatively insensitive to the size of the nose radius. These results are based on entries into an air atmosphere. The heating rates along the body are nondimensionalized by the stagnation-point value, $q_w^R/q_{w,o}^R$, and the distance along the body is nondimensionalized by the nose radius, s/Rn .

The present method was used to investigate the above mentioned trends. The results presented are for spherically-capped, conical bodies since this is the body shape presently being considered for planetary entries. However, the present method is not restricted to this type of body shape. Results from stagnation-point solutions are also presented.

Results of radiative heating distributions from solutions for entries into different atmospheres and different entry conditions are presented in Figure 9. The results for the two entry conditions of the CO_2-N_2 mixture are representative of entry to Venus and the results for the H_2-He mixture are representative of entry to Jupiter. Note that the distributions are highly nonsimilar for the different cases. The results for case C and D are for the same atmosphere but different entry conditions; but, the distributions are nonsimilar. The present method can be readily used for entries into atmospheres other than air.

A comparison of the stagnation-point, radiative heating rates for entries in air and in a $.90CO_2-.10N_2$ mixture (by volume) is presented

in Figure 10. The radiative heating in the $\text{CO}_2\text{-N}_2$ mixture is much greater than in air at the lower velocities; but, the results are comparable at the higher velocities. The difference at the lower velocities is due to the molecular band radiation from the red and violet bands of cyanogen, CN(R) and CN(V) , and the fourth positive band of carbon dioxide, CO (4+) , in the $\text{CO}_2\text{-N}_2$ mixture. The atomic species are dominant at the higher velocities, greater temperature behind the shock wave, and the radiation from the continuum and line transitions is greater than from the molecular band systems.

As the radiating gas flows around a body there are two effects which can influence the radiative heating to the body. The volume of radiating gas increases, increased shock stand-off distance, which will increase the radiative heating. This effect is offset by the decrease in temperature as the gas expands around the body. The distributions of radiative heating along spherically-capped, conical bodies for entry in an air atmosphere are presented in Figure 11 for different body angles. The distributions are nonsimilar and the nondimensional radiative heating is greatest for the largest body angle. The shock stand-off distances and temperatures at a location on the conical portion of the body are also presented. Both the temperature and shock stand-off distance are greater for the larger body angle; thus, the greater the radiative heating. Note that the dimensional, stagnation-point heating rate is insensitive to the downstream body angle for spherically-capped, conical bodies. These results are in agreement with the trends presented by Callis (3) and Olstad (4).

The effect of body angle on the nondimensional, radiative heating distribution for entries to Venus is presented in Figure 12 for two entry conditions. As for air entry, the radiative heating rates along the conical region of the body are greater for the larger cone angle. Also, the stagnation-point, radiative heating rate is insensitive to the cone angle. As previously stated, the nondimensional, radiative heating distribution is nonsimilar with respect to entry conditions. For the 60° body, the radiative heating rates along the downstream region of the body exceed the stagnation-point value at the lower velocity entry. At the higher velocity entry, the radiative heating rate increases along the downstream region but does not exceed the stagnation-point value.

The effect of nose radius on the nondimensional, radiative heating distributions for an entry in air is presented in Figure 13. The distributions are slightly nonsimilar for the downstream region of the body. Callis (ref. 3) and Olstad (ref. 2) state the distributions to be nonsimilar and the present results are in agreement. However, the results of Callis and Olstad indicate a greater nonsimilarity than shown by the present results for an air entry. The reason is probably due to the radiation model used by Callis and Olstad which might be too transparent as previously discussed. In which case, the nose radius would have a greater effect than shown by the present results. Suttles (ref. 89) states the distributions along the subsonic flow region of the body to be similar with respect to nose radius. The present results are in agreement with Suttles since the distributions are similar for the forward region of the body, $s/R_n < 0.4$.

The effect of nose radius on the nondimensional, radiative heating distributions for entries to Venus is presented in Figure 14. The distributions are slightly nonsimilar for the downstream region of the body for the lower velocity entry. However, the distributions are highly nonsimilar for the higher velocity entry. For both entries, the distributions are similar at the forward region of the body up to approximately the sphere-cone junction.

The present results are based on a limited range of entry conditions and body shapes. This same situation applies to the studies by Callis (ref. 3), Olstad (ref. 2), and Suttles (ref. 89) for entries only in an air atmosphere. Based on the present results, the following trends seem to be generally prevalent. The nondimensional, radiative heating distributions ($q_w^R/q_{w,0}^R$ versus s/Rn) are nonsimilar with respect to entries in different atmospheres and with respect to entries in the same atmosphere at different entry conditions. The distributions are also nonsimilar with respect to body angle and nose radius. Extreme caution should be exercised in attempting to extrapolate the results from known distributions to other entry conditions or body shapes for which solutions have not yet been obtained. For large body angles, the radiative heating rates to the downstream region of a body can exceed the stagnation point value for certain entry conditions.

The exact composition of the Venusian atmosphere is unknown at present. The most recent data indicates the composition to be predominately CO_2 with up to 10 percent by volume of N_2 . The effect of gas

composition for $\text{CO}_2\text{-N}_2$ mixtures on the radiative heating to the stagnation-point of a body is presented in Figure 15 for two entry conditions. For the higher velocity entry, the radiative heating rate varies by only 25 percent over the range of CO_2 content with the higher values occurring for the greater percentage of CO_2 . For the lower velocity entry, the radiative heating rate can vary by a factor of 6 over the range of CO_2 content. This is due to the domination of radiation by the molecular band transitions of $\text{CO}(4+)$, $\text{CN}(V)$, and $\text{CN}(R)$ at the lower velocity. For the composition of present interest for Venusian entry (less than 10 percent N_2), the effect of composition on the stagnation-point, radiative heating rate is negligible at the higher velocity; but, a 22 percent difference occurs with composition at the lower velocity.

Fully-Coupled, Stagnation Point, Solutions for Earth Reentry

The present method has been used to obtain fully-coupled solutions at the stagnation point of blunt bodies for Earth Reentry. The solutions are for mass injection at the wall of air and ablation products. The results from the present method are compared with the results given by Garrett (ref. 19).

The solutions were obtained for the free-stream conditions and mass injection rates presented in Table II. These conditions are representative of a manned spacecraft reentering the Earth's atmosphere after exploration of Mars. For the solutions, the wall conditions of the temperature and the rate and composition of the injected gas were prespecified. This procedure was used to be consistent with the method of

Garrett. The results from the present method and the results given by Garrett are presented in Table II.

The present results agree with the results of Garrett within 5 percent for the mass injection of ablation products and within 10 percent for the mass injection of air. This good agreement is better than has been previously shown in comparing any two methods (see ref. 19). As shown by the results, the mass injection of ablation products is twice as effective as the mass injection of air in reducing the radiative heating to the wall. The mass injection of ablation products reduced the radiative heating to the wall by 40 percent at the larger blowing rate.

The present results for the temperature profile and the radiative flux profile through the gas layer for a solution are compared with the results of Garrett in Figure 16. The specified conditions are given in Figure 16(a) and the temperature profile and radiative flux profile are given in Figure 16(b) and 16(c), respectively. The present results are in good agreement with the results of Garrett. For the present solution, spectral distributions of radiative heating toward the wall due to continuum processes and due to line processes are presented in Figures 16(d) and 16(e), respectively. Spectral distributions were not given in the thesis by Garrett.

For the continuum processes, practically all radiation beyond 11 eV is absorbed within the boundary layer and is attributed to the atomic species being at a lower temperature in the boundary layer than in the inviscid layer. There is a slight absorption of radiation within the boundary layer between the spectral range of 4 to 10 eV but the effect

is offset by increased radiation emission in the spectral range of 1 to 4 eV. For the line processes, the radiation is sharply reduced in the spectral region of 7 to 11 eV. Part of this reduction for the line groups at 7.1, 8.4 and 9.4 eV is due to the absorption by the fourth positive band system of carbon monoxide, $\text{CO}(4+)$. The carbon monoxide is present in the boundary layer from the mass injection of ablation products. The present results for the spectral regions of absorption, or increased emission, within the boundary layer are in agreement with the results given in reference 28.

The good agreement of the present results with those given by Garrett (ref. 19) shows that the present method for a fully-coupled solution with ablation product effect provides a good solution at the stagnation point of a blunt body for Earth reentry. The method by Garrett can only be used at the stagnation point of a blunt body and has not been used for entries in atmospheres other than air.

Fully-Coupled Solutions for Venusian Entry

The present method was used to obtain solutions at typical conditions for unmanned, scientific probes during Venusian entry. One mission currently under study is a multi-probe entry with one large probe and three small probes which are spherically-capped, conical bodies. A description of this type of mission is presented in references 61 to 64. The entry probes are released from a spacecraft and are targeted to different locations on Venus.

The entry parameters and body shapes listed below were used to calculate nominal trajectories for use in the present analysis. The only small probe considered herein is the one entering at the steepest entry angle.

| | <u>Large Probe</u> | <u>Small Probe</u> |
|-----------------------------|--------------------|--------------------|
| Entry velocity, km/s | 11.06 | 11.06 |
| Entry angle, degrees | -42 | -56 |
| $W/C_d A$, kg/m^2 | 78.5 | 125.7 |
| Weight, kg | 158.8 | 22.7 |
| Nose radius, m | .3429 | .1397 |
| Body half-angle, degrees | 60 | 45 |
| Base diameter, m | 1.3716 | .4064 |

The atmospheric model used for the Venusian atmosphere is that presented in reference 62 and the gas composition is 97 percent carbon dioxide and 3 percent nitrogen by volume. The calculated entry trajectories are presented in Figure 17.

The radiative heating rates at the stagnation point of the bodies were calculated along the trajectories to determine the location of peak radiative heating. The "stagnation-point" version of the developed, radiating inviscid flow program was used for these calculations. The results are presented in Figure 18 for the large and small probes. Even though the nose radius is smaller, the radiative heating to the small probe is greater due to the deeper penetration in the denser regions of the atmosphere at higher velocities. Solutions for the radiating, inviscid flow around the bodies were then obtained at the peak heating conditions and the stagnation-point values of radiative heating rates from these solutions are also presented in Figure 18. It is at these conditions for peak radiating heating that fully-coupled solutions with

ablation product effects were obtained for the radiating gas flow around the large and small probes. Hereafter, the solutions are referred to as large and small probes.

The ablation of the heatshield for the fully-coupled solution is represented by steady-state ablation of a carbon-phenolic material with elemental mass fractions of carbon, 0.851; oxygen, 0.110; hydrogen, 0.035; and nitrogen, 0.004. The ablation rates are solved for as part of the fully-coupled solution and are not prespecified.

The results from the fully-coupled solutions for the large and small probes are presented in Figures 19 and 20, respectively. The distribution along the body of radiative heating, convective heating, ablation rate, aerodynamic shear, and momentum thickness Reynolds number are presented. The spectral distribution of radiative heat flux due to continuum processes is also presented for two body locations.

For the large probe, the ablation rate decreases along the body in the nose region and reaches a nearly constant value along the afterbody (Figure 19(d)). As shown in Figure 19(c), the convective heating rate decreases along the afterbody but the radiative heating rate increases to values which exceed the stagnation-point value. The radiative heating to the wall is greater than the convective heating along the entire body. The total heating rate, radiative plus convective, along the afterbody is 30 to 40 percent less than the stagnation-point value. Thus, the decrease in convective heating rate along the afterbody negates the effect of the increase in radiative heating rate. The wall temperature resulting from the steady-state ablation is 3600°K , within 2 percent, along the body.

For the small probe, there is a decrease in the ablation rate along the afterbody (Figure 20(d)) and both the convective and radiative heating rates decrease along the afterbody (Figure 20(c)). The convective heating rate is greater than the radiative heating rate along the entire body. This result is opposite to the result obtained for the large probe. The total heating rate along the afterbody of the small probe is 40 to 60 percent less than the stagnation-point value. The wall temperature resulting from the steady-state ablation is 3700°K , within 2 percent, along the body.

The radiative heating rates from inviscid, radiating solutions are compared with the results for the fully-coupled solutions with steady-state ablation in Figures 19(b) and 20(b) for the large and small probes, respectively. The radiative heating rates from the fully-coupled solutions are less than the values from the inviscid, radiating solutions by 9 and 17 percent, respectively, for the large and small probes at the stagnation point of the bodies. These percent reductions are nearly constant along the bodies. The reduction in radiative heating is due to absorption of radiation within the boundary layer.

Spectral distributions of radiative heating toward the body due to continuum processes are presented in Figures 19(g) and 20(g), respectively, for the large and small probes. The spectral distributions are presented for the stagnation point and an afterbody location. As shown by the results, the absorption of radiation within the boundary layer occurs in the spectral range of 5 to 10 eV and is due to self absorption

of the fourth positive band system of carbon monoxide, $\text{CO}(4+)$. At the stagnation point of the small probe there is a slight increase in radiative heating within the boundary layer at 2.5 eV and is due to the Swan band system of diatomic carbon, $\text{C}_2(\text{S})$. A similar effect was noted at the other body locations for both the large and small probes, but the effect was too small to be presented in the figures. The radiative heating due to line processes was less than ten percent of the total radiative heating at these entry conditions and spectral distributions are not presented.

The fully-coupled solutions assumed laminar flow in the boundary layer. Shown in Figures 19(f) and 20(f) are the distributions of momentum thickness Reynolds number around the bodies. If the critical value for transition to turbulent flow is about 200, then the boundary layer would be turbulent along the afterbody for both the large and small probes. This could have an appreciable effect on the distribution of convective heating, ablation rate, and aerodynamic shear.

A fully-coupled solution for the large probe with a turbulent boundary layer was obtained and the results are compared with the laminar case in Figure 21. A critical value of 200 for the momentum thickness Reynolds number was used for transition to turbulent flow. As expected, the convective heating rates, ablation rates, and aerodynamic shears increased with transition to turbulent flow. The distribution along the body of radiative heating was essentially the same for both laminar and turbulent flow; thus, the turbulent boundary layer did not appear to further attenuate the radiative heating even though the

boundary-layer thickness was 80 percent greater than for laminar flow. This phenomenon can be explained with the wall spectral distributions presented in Figure 21(d) for the turbulent and laminar solutions. For the turbulent boundary layer, there is an increase in the self absorption of the CO(4+) band system in the spectral range of 5 to 8 eV but this effect is offset by an increased emission of the C₂(S) band system at 2.5 eV and the red band system of cyanogen, CN(R), at .6 to 2 eV. The spectral distribution at the boundary layer edge is essentially the same for both the turbulent and laminar boundary layers and was presented in Figure 19(g). While the radiative heating rates along the afterbody of the large probe are the same for a turbulent or a laminar boundary layer, the wall spectral distribution of the radiation is different for the two cases. It should not be inferred that the radiative heating to the body will be the same for a laminar or turbulent boundary layer for other entry conditions or body shapes.

A fully-coupled solution was obtained for free-stream conditions representative of a higher entry velocity than the nominal trajectories and the results are presented in Figure 22. The free-stream conditions and body shape are given in Figure 22(a). The body shape is the same as the large probe except the nose radius is slightly smaller. Hereafter, this case is referred to as the high velocity entry.

As shown in Figure 22(c), the radiative heating rate is greater than the convective heating rate along the entire body. Unlike the large probe, there is only a slight increase in the radiative heating along the afterbody. The convective heating rate at the stagnation

point is slightly less than the values just downstream of the stagnation point and is due to an increased effect of blockage of convective heating by the mass injection of the ablation products. As shown in Figure 22(d), the ablation rate decreases along the body in the nose region. The ablation rate and total heating rate are nearly constant along the afterbody. The wall temperature resulting from the steady-state ablation is 3750°K , within 1 percent, along the body.

There is significant absorption of radiation within the boundary layer for the high velocity entry as shown by a comparison of results between the fully-coupled solution and an inviscid, radiating solution in Figure 22(b). The reduction in radiative heating rate is 25 percent at the stagnation point and 25 to 30 percent along the afterbody. Spectral distributions of radiative heating toward the body due to continuum processes are presented in Figure 22(g) for the stagnation point and an afterbody location. For the continuum processes, most of the absorption within the boundary layer occurs in the spectral region of 5 to 10 eV and is due mainly to self absorption of the $\text{CO}(4+)$ band. There is an increase in radiative flux at 2.5 eV and is due to the $\text{C}_2(\text{S})$ band. At the stagnation point there is absorption at photon energies greater than 10 eV and this absorption is attributed to atomic species within the cooler regions of the boundary layer. There is also absorption at the photoionization edge of carbon at 8.5 eV.

The radiative flux due to line processes is 50 percent of the total radiative flux at the nose region of the body and decreases to only 20 percent along the afterbody. Spectral distributions of the radiative

heating due to line processes for two locations at the nose region are presented in Figure 22(h). Absorption of line radiation within the boundary layer occurs only for the line groups in the ultraviolet at 7.1, 8.4, and 9.4 eV. The line radiation in the visible and infrared regions of the spectral, less than 2 eV, is not attenuated within the boundary layer.

The present results have shown that the radiative heating toward the body is attenuated in the boundary layer for Venusian entries. This attenuation will reduce the radiative heating to the body by 10 to 20 percent at entry conditions and body shapes which are presently considered as nominal. The reduction can be as large as 30 percent at higher velocity entries. The radiative heating toward the body is attenuated in the boundary layer at the downstream region of the body as well as at the stagnation point of the body.

SUMMARY AND CONCLUSIONS

A method is presented for the solution of the fully-coupled, nongray radiating gas flow about an ablating, planetary entry body. The solution is for a gas in chemical equilibrium and arbitrary gas mixtures can be considered. The treatment of radiation accounts for molecular band, continuum, and atomic line transitions with a detailed frequency dependence of the absorption coefficient. The ablation of the entry body is solved as part of the solution for a steady-state ablation process.

Application of the developed method is shown by results at typical conditions for unmanned, scientific probes during entry to Venus. The radiative heating toward the body is attenuated in the boundary layer for Venusian entries. This attenuation will reduce the radiative heating to the body by 10 to 20 percent at entry conditions and body shapes which are presently considered as nominal. The reduction can be as large as 30 percent at higher velocity entries. The attenuation of radiation within the boundary layer is primarily due to the self absorption of the fourth positive band system of carbon monoxide, $\text{CO}(4+)$, in the radiation spectral region of 5 to 10 eV. At high velocity entry, the attenuation of radiation also occurs for line transitions in the radiation spectral region of 7 to 10 eV.

Prior studies of fully-coupled solutions with ablation product effects have been for the stagnation point of a body for Earth reentry and have shown that the boundary layer with injection of ablation products is effective in reducing the radiative heating to the body. The

present results show that the boundary layer over an ablating body will reduce the radiative heating to the body for entries to Venus. Furthermore, the attenuation of radiation within the boundary layer occurs at downstream regions of the body as well as the stagnation point of the body.

Present results from a study of the radiating, inviscid flow about spherically-capped, conical bodies during planetary entries show that the nondimensional, radiative heating distributions ($q_w^R/q_{w,0}^R$ versus s/R_n) to be nonsimilar with respect to entries in different atmospheres and with respect to entries in the same atmosphere at different entry conditions. The distributions are also nonsimilar with respect to nose radius and downstream body angle. Therefore, extreme caution should be exercised in attempting to extrapolate the results from known distributions to other entry conditions for which solutions have not yet been obtained. The radiative heating rates to the downstream region of the body can exceed the stagnation-point value for certain entry conditions and body shapes.

EXTENSION OF PRESENT RESEARCH

An immediate extension of the present research can be made in the following areas:

1. Additional fully-coupled solutions are needed for a better evaluation of the thermal environment for Venusian entries. Solutions with a turbulent boundary layer are needed for the small probe and for higher velocity entries.
2. Solutions are needed for body shapes other than spherically-capped, conical bodies for Venusian entries. Future design may dictate a different shape and the results would be useful in evaluating the effects of such a change in shape.
3. A study is needed for the effect of composition for $\text{CO}_2\text{-N}_2$ mixtures on the radiative heating distribution along a body for Venusian entries.
4. Use the present results and results from additional solutions to develop correlations which can be easily used in parametric design studies for Venusian entries.
5. The present computational method needs to be extended to include a transient ablation analysis.
6. Use the present method to begin studies of entries to Saturn. Saturn is characteristic of the giant planets (Jupiter, Saturn, Uranus, and Neptune) and probably will be the first chosen for exploration by entry probes.

TABLE I. - STAGNATION-POINT, RADIATIVE HEATING
RATES FOR AIR ENTRY.

$$V_{\infty} = 14.55 \text{ km/s}$$

$$\rho_{\infty} = 2.252 \times 10^{-4} \text{ kg/m}^3$$

$$p_{\infty} = 16.16 \text{ N/m}^2$$

| Method | $q_{w,o}^R, \text{ MW/m}^2$ | |
|-------------------------------|-----------------------------|--------------|
| | Rn = .3427 m | Rn = 3.427 m |
| Present method | 16.17 | 28.80 |
| Callis, Ref. 3 | 18.71 | 45.47 |
| Suttles, Ref. 89 | 18.38 | 27.26 |
| Falanga and Sullivan, Ref. 73 | 18.73 | 28.44 |

TABLE II. - RESULTS FROM FULLY-COUPLED, STAGNATION-POINT
SOLUTIONS WITH MASS INJECTION FOR AIR ENTRIES.

(a) $V_{\infty} = 15.25 \text{ km/s}$; $\rho_{\infty} = 2.72 \times 10^{-4} \text{ kg/m}^3$; $p_{\infty} = 17.76 \text{ N/m}^2$
 $Rn = 3.048 \text{ m}$; $\bar{T}_w = 3600^{\circ} \text{ K}$

| Injected gas | | $q_{w,o}^R, \text{ MW/m}^2$ | |
|---------------------------------------|--------------------|-----------------------------|------------------|
| $(\rho v)_w, \text{ kg/m}^2\text{-s}$ | Composition | Present method | Garrett, Ref. 19 |
| 0 | - | 3980 | 4150 |
| .415 | air | 3375 | 3740 |
| .830 | air | 3090 | 3490 |
| .415 | ablation products* | 2870 | 2770 |
| .830 | ablation products* | 2560 | 2460 |

* $\bar{K}_C = .9207$; $\bar{K}_O = .0491$; $\bar{K}_H = .0216$; $\bar{K}_N = .0086$

(b) $V_{\infty} = 15.25 \text{ km/s}$; $\rho_{\infty} = 1.77 \times 10^{-4} \text{ kg/m}^3$; $p_{\infty} = 12.26 \text{ N/m}^2$
 $Rn = 2.56 \text{ m}$; $\bar{T}_w = 3840^{\circ} \text{ K}$

| Injected gas | | $q_{w,o}^R, \text{ MW/m}^2$ | |
|---------------------------------------|--------------------|-----------------------------|------------------|
| $(\rho v)_w, \text{ kg/m}^2\text{-s}$ | Composition | Present method | Garrett, Ref. 19 |
| 0 | - | 2307 | 2540 |
| .205 | ablation products* | 1710 | 1620 |

* $\bar{K}_C = .923$; $\bar{K}_O = .058$; $\bar{K}_H = .018$; $\bar{K}_N = .001$

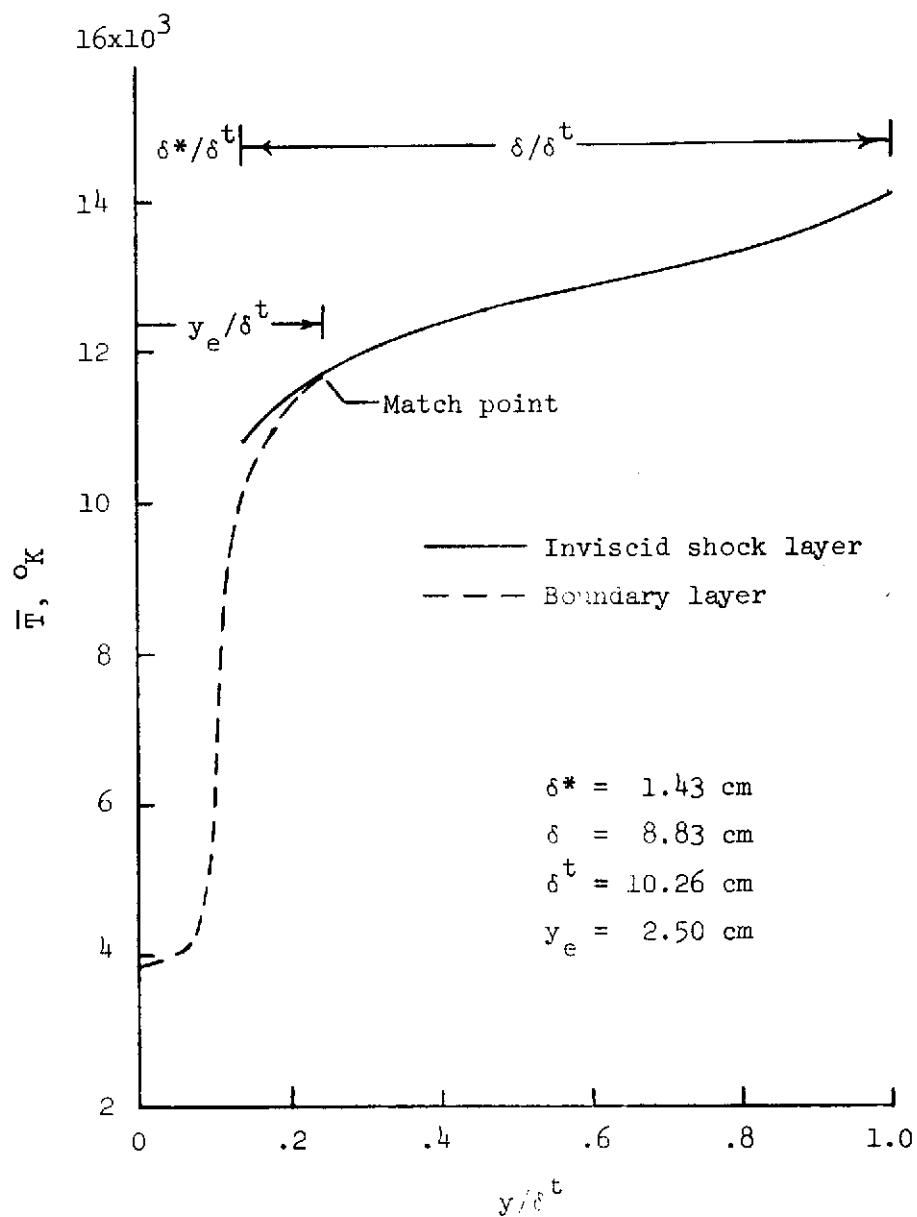


Figure 1. - Illustration of the mating of the boundary layer solution to the inviscid layer solution.

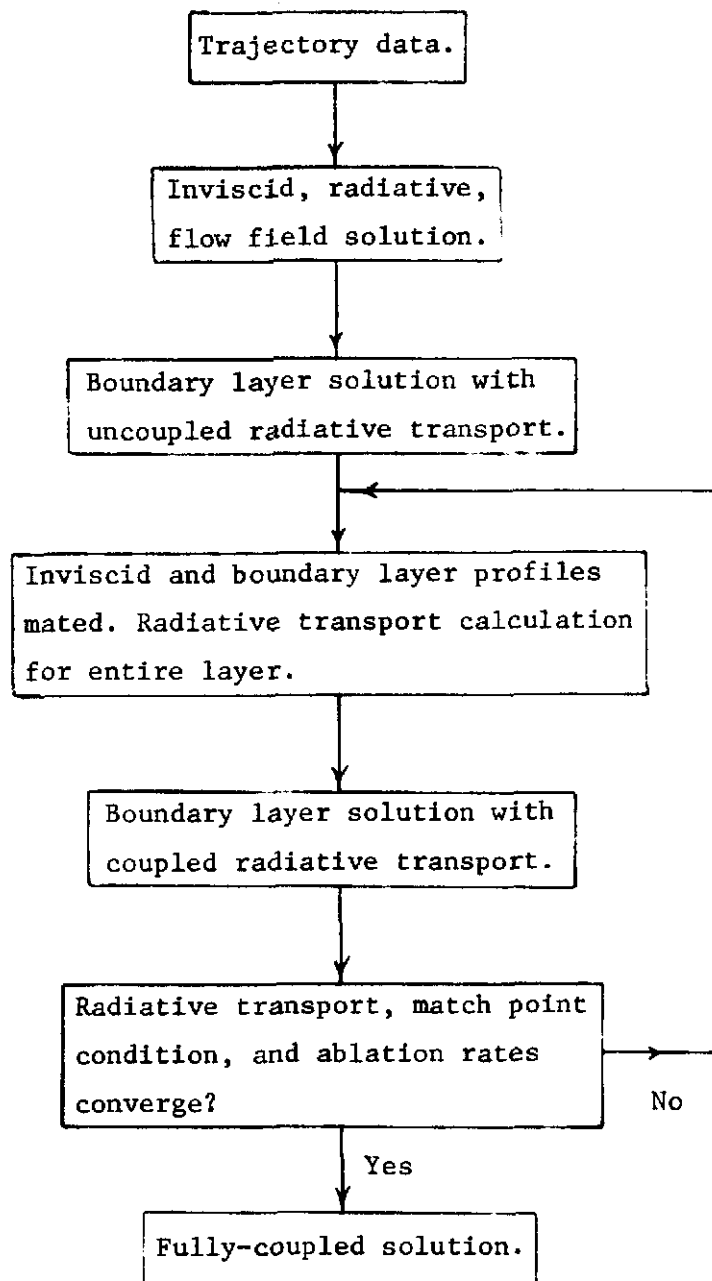
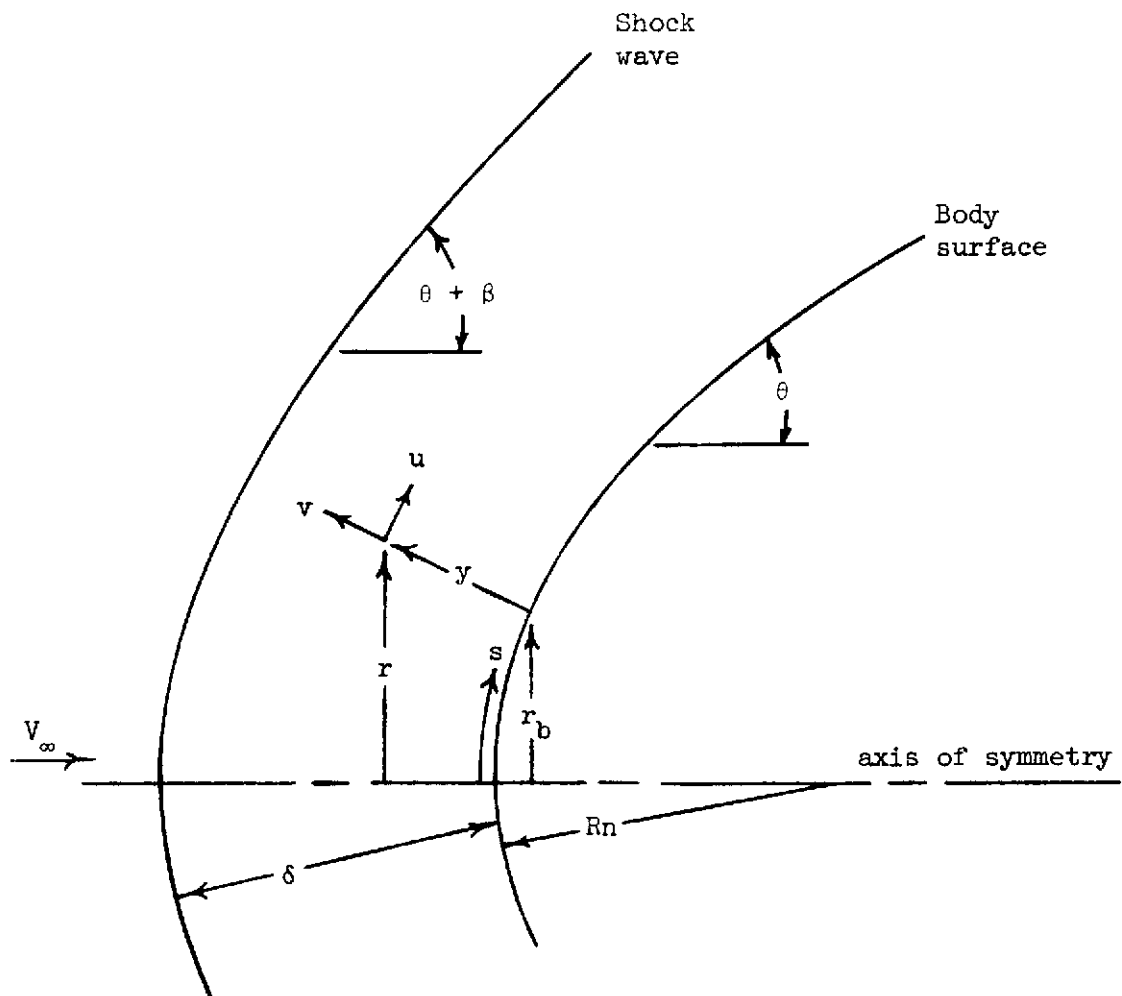


Figure 2. - Calculation procedure for the fully-coupled, radiating, flow field solution.



Coordinates

- s - Distance along body surface from axis of symmetry.
- y - Distance perpendicular to body surface.
- ϕ - Angle measured in plane normal to axis of symmetry.

Figure 3. - Flow field coordinate system for axisymmetric blunt body.

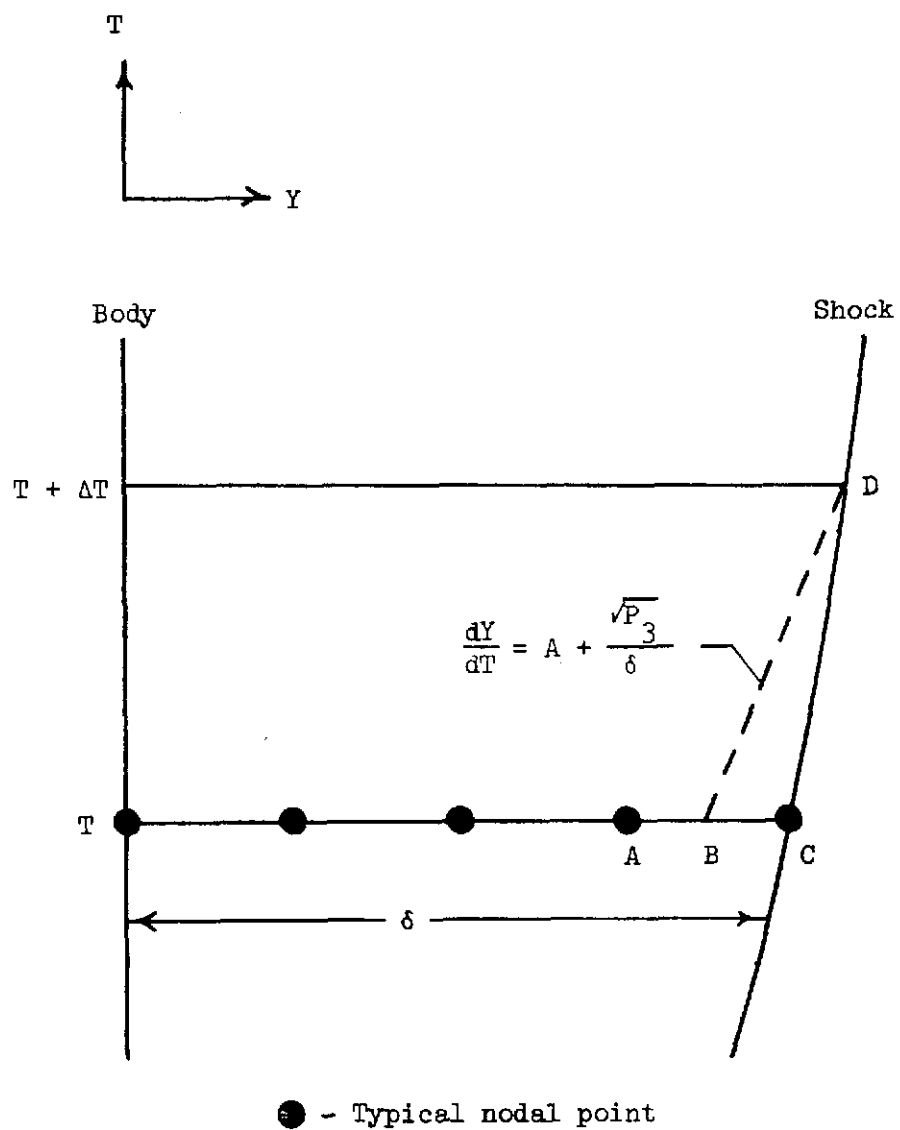
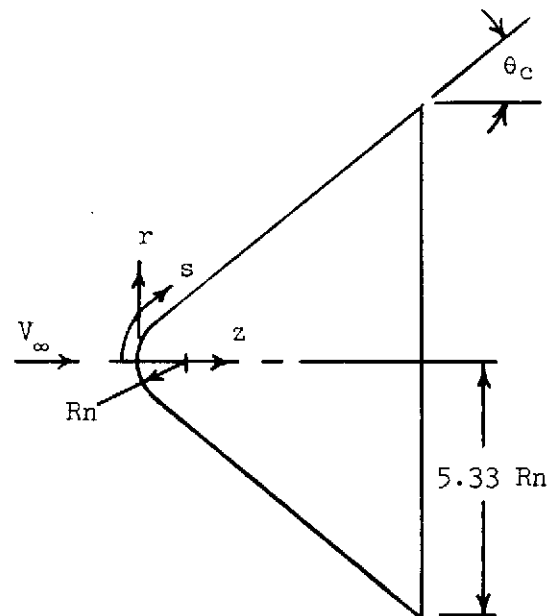


Figure 4. - Schematic of use of unsteady characteristics at shock wave.



Gas composition by volume

.79 N_2 - .21 O_2 (air)

$V_\infty = 12.192$ km/s

$\rho_\infty = 3.15 \times 10^{-4}$ kg/m³

$p_\infty = 22.6$ N/m²

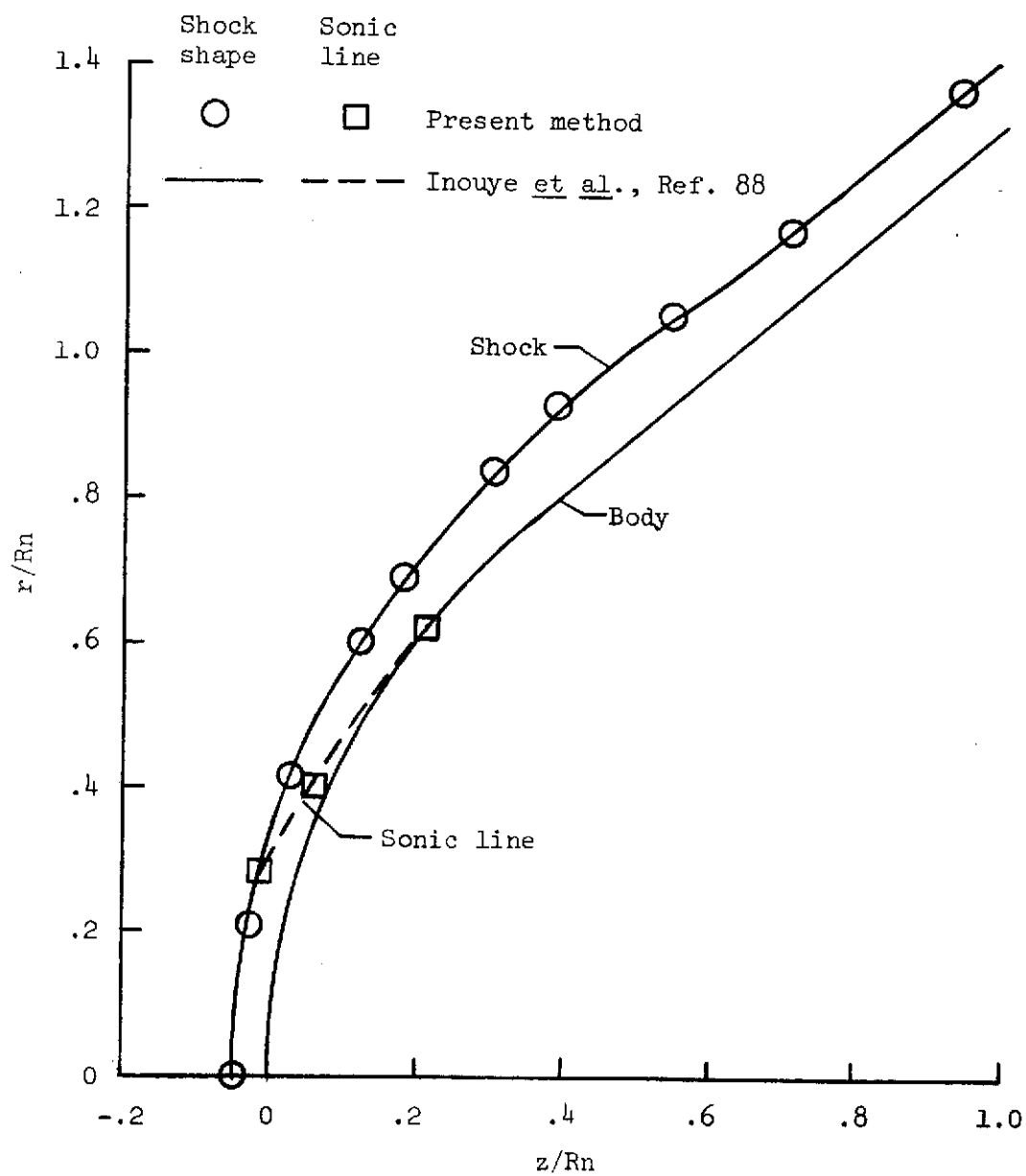
$h_\infty = 0.0$ J/kg

$R_n = 2.0$ m

$\theta_c = 40^\circ$

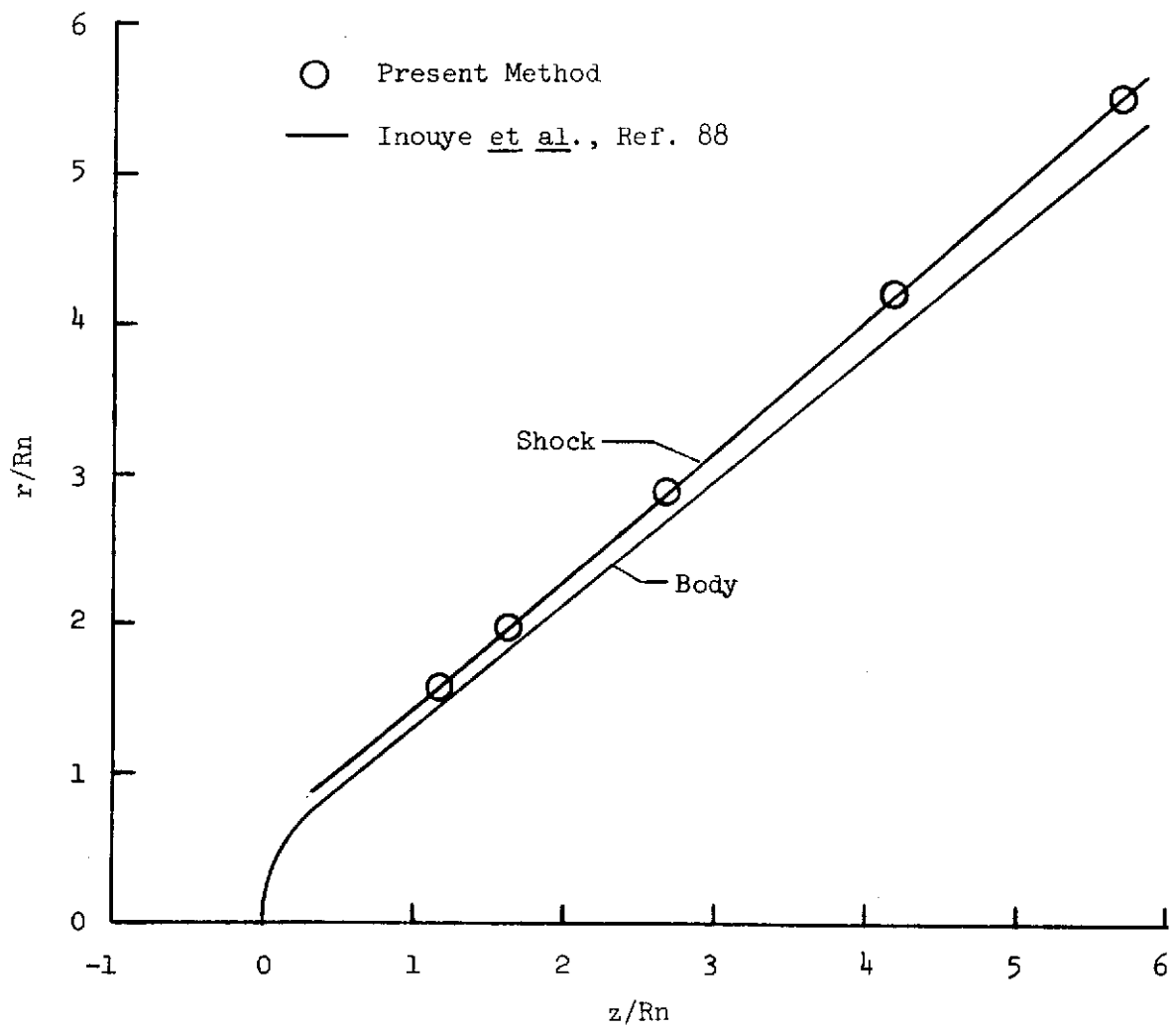
(a) Free-stream conditions and body shape.

Figure 5. - Nonradiating, inviscid flow solution around a blunt body. Present method is compared with the method of Inouye et al. (ref. 88).



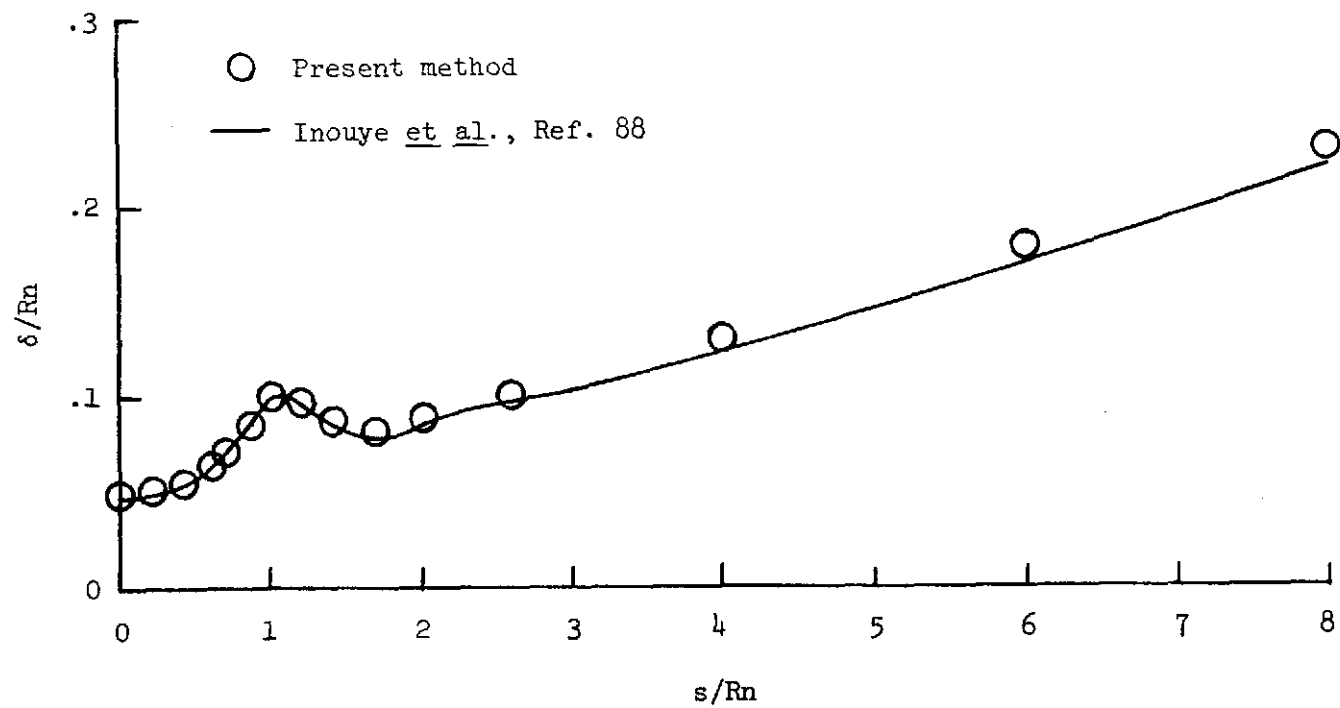
(b) Shock shape at forward region of body and sonic line location.

Figure 5. - Continued.



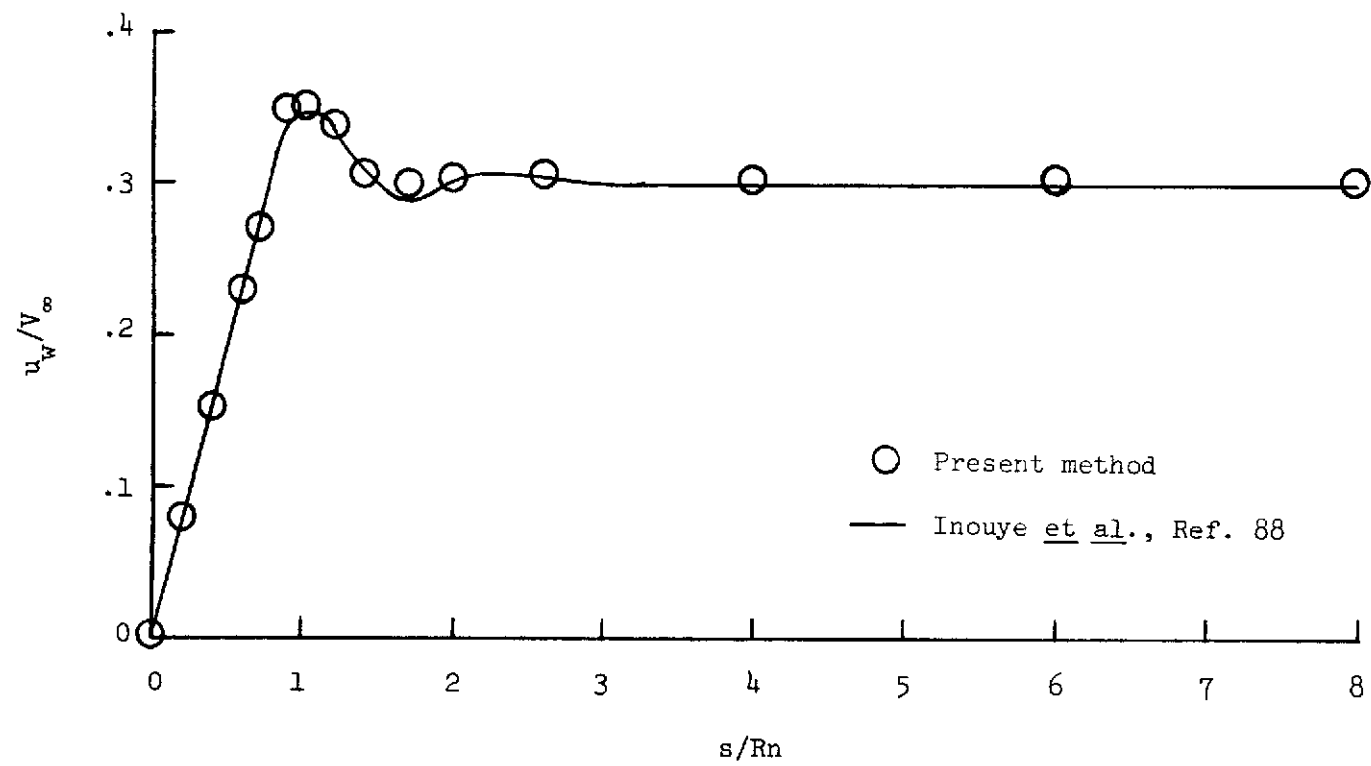
(c) Shock shape at downstream region of body.

Figure 5. - Continued.



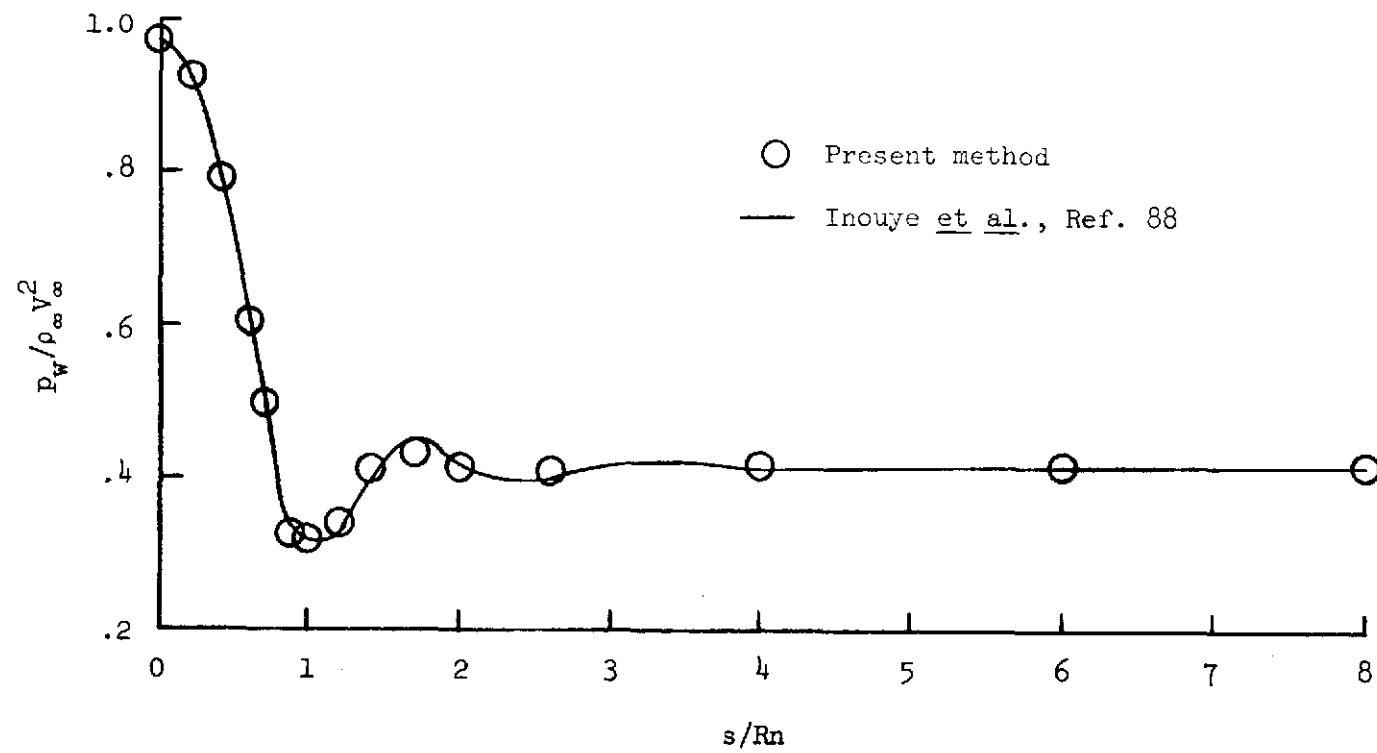
(d) Shock stand-off distance around body.

Figure 5. - Continued.



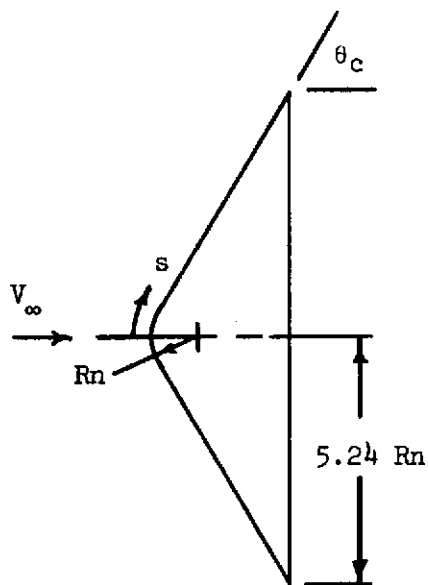
(e) Velocity distribution along body.

Figure 5. - Continued.



(f) Pressure distribution along body.

Figure 5. - Concluded.



Gas composition by volume

.79 N_2 - .21 O_2 (air)

$V_\infty = 12.20$ km/s

$\rho_\infty = 2.74 \times 10^{-4}$ kg/m³

$p_\infty = 20.0$ N/m²

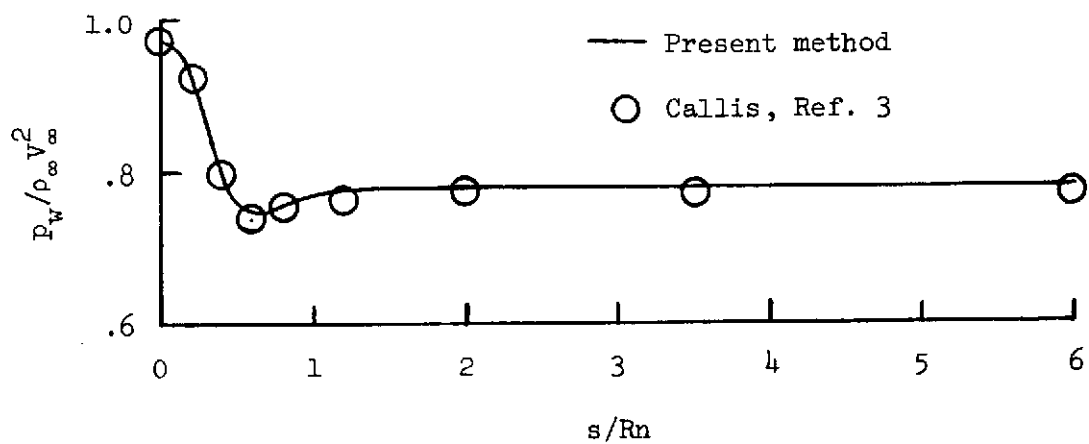
$h_\infty = 0.0$ J/kg

$R_n = 0.20$ m

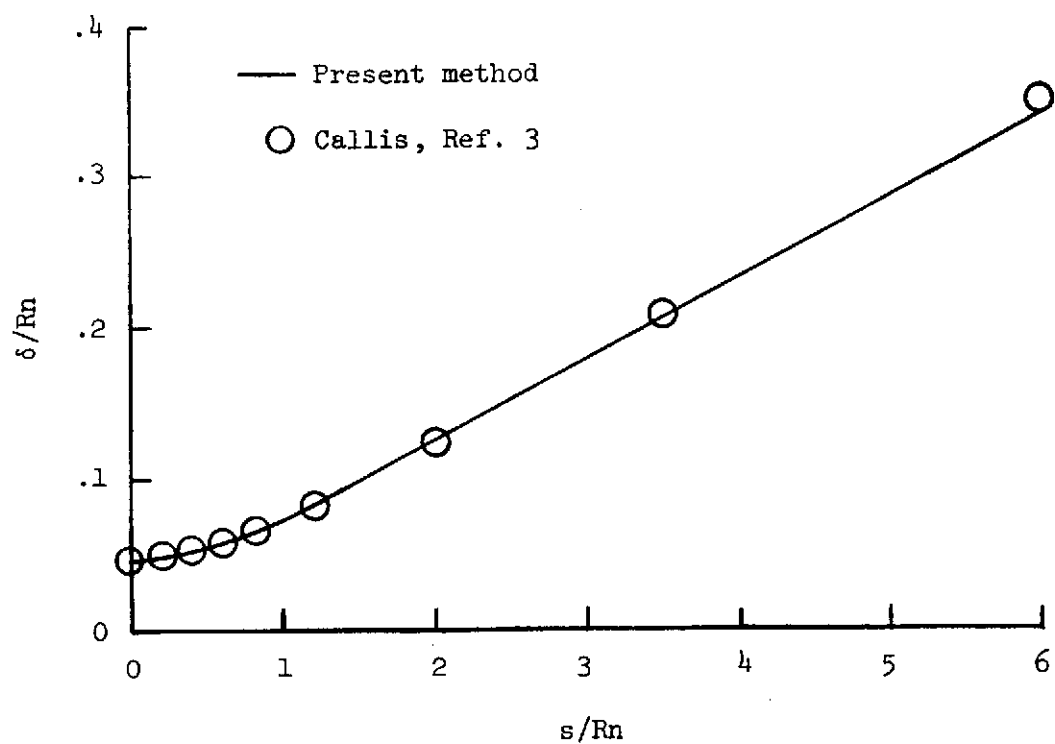
$\theta_c = 60^\circ$

(a) Free-stream conditions and body shape.

Figure 6. - Radiating, inviscid flow solution about a blunt body during Earth reentry. Present method is compared with the method of Callis (ref. 3).

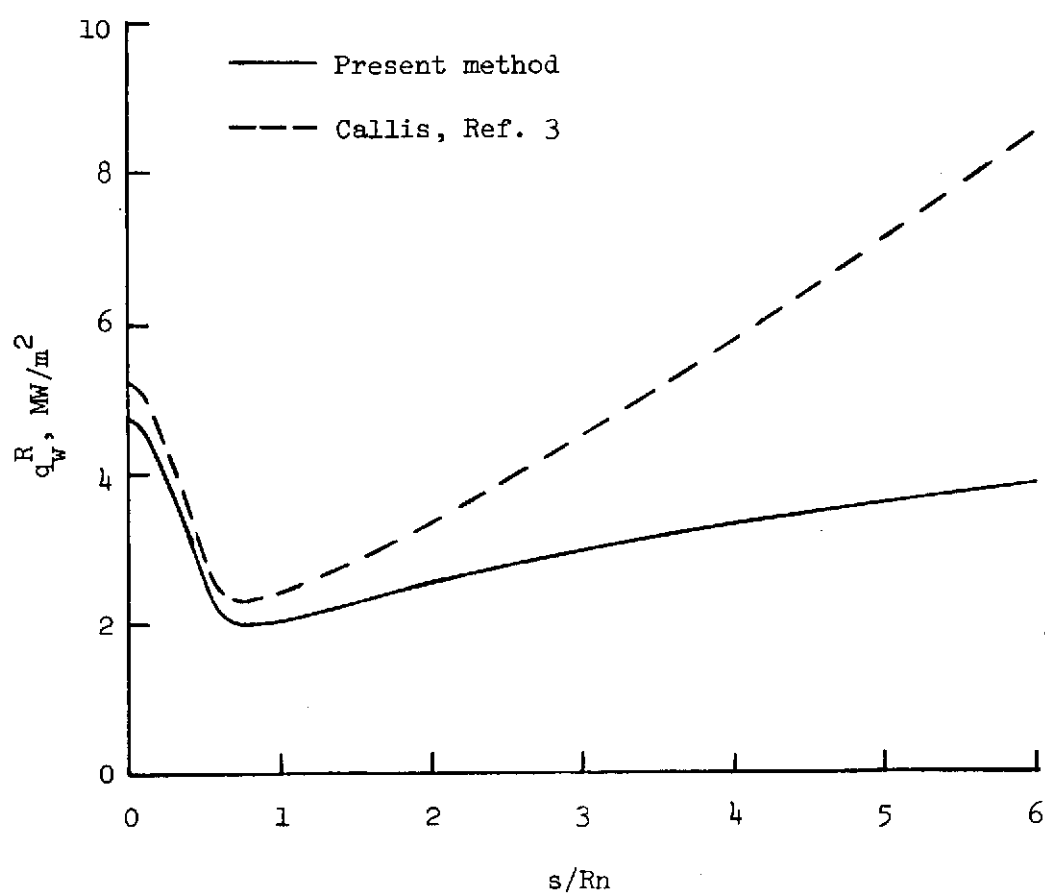


(b) Pressure distribution along body.



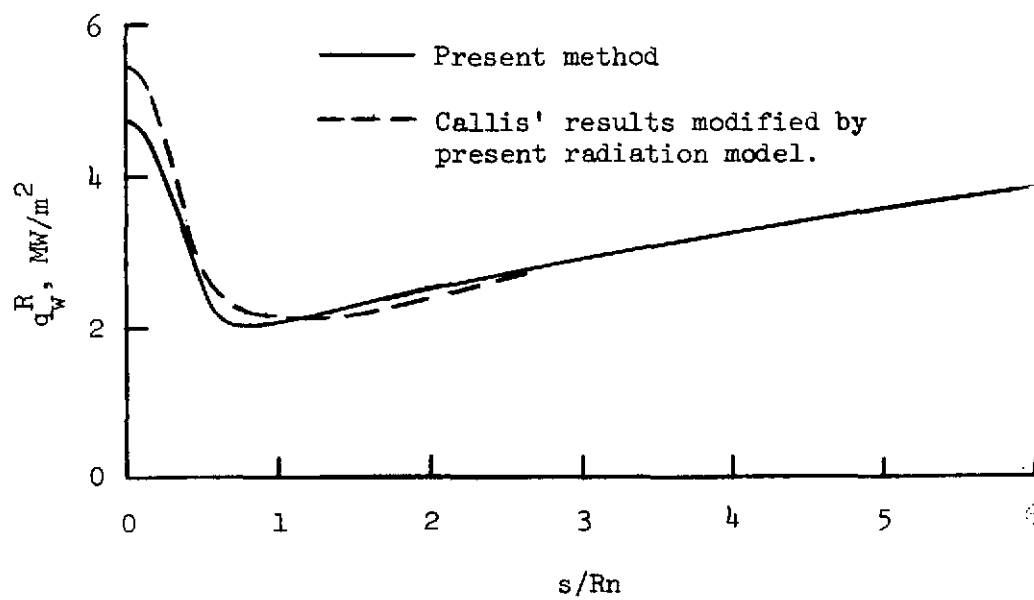
(c) Shock stand-off distance around body.

Figure 6. - Continued.



(d) Radiative heating along body.

Figure 6. - Continued.



(e) Radiative heating along body.
Callis' results modified.

Figure 6. - Concluded.

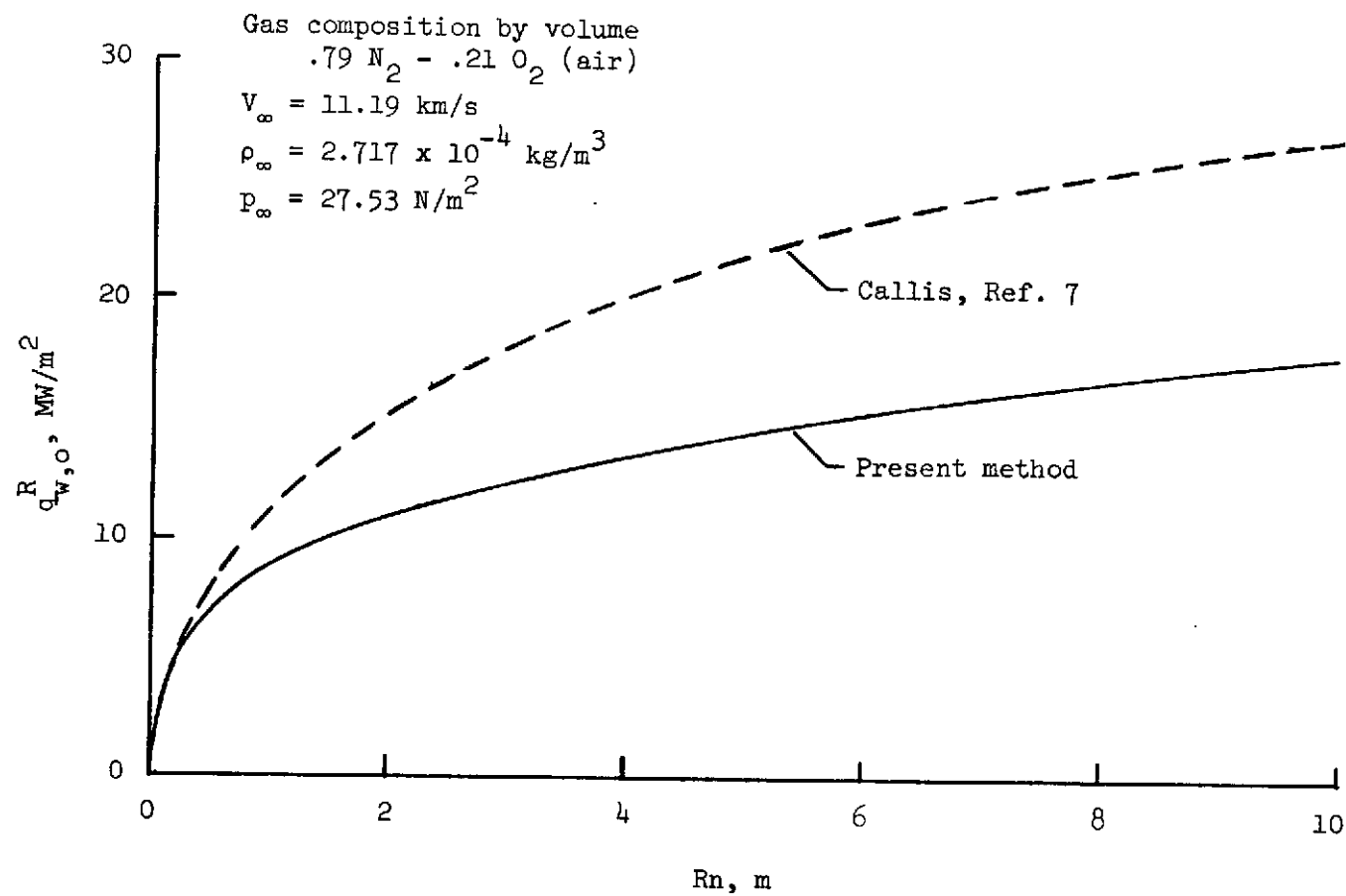


Figure 7. - Stagnation-point, radiative heating rates in an air atmosphere as a function of nose radius. Present results are compared with the method of Callis (ref. 7).

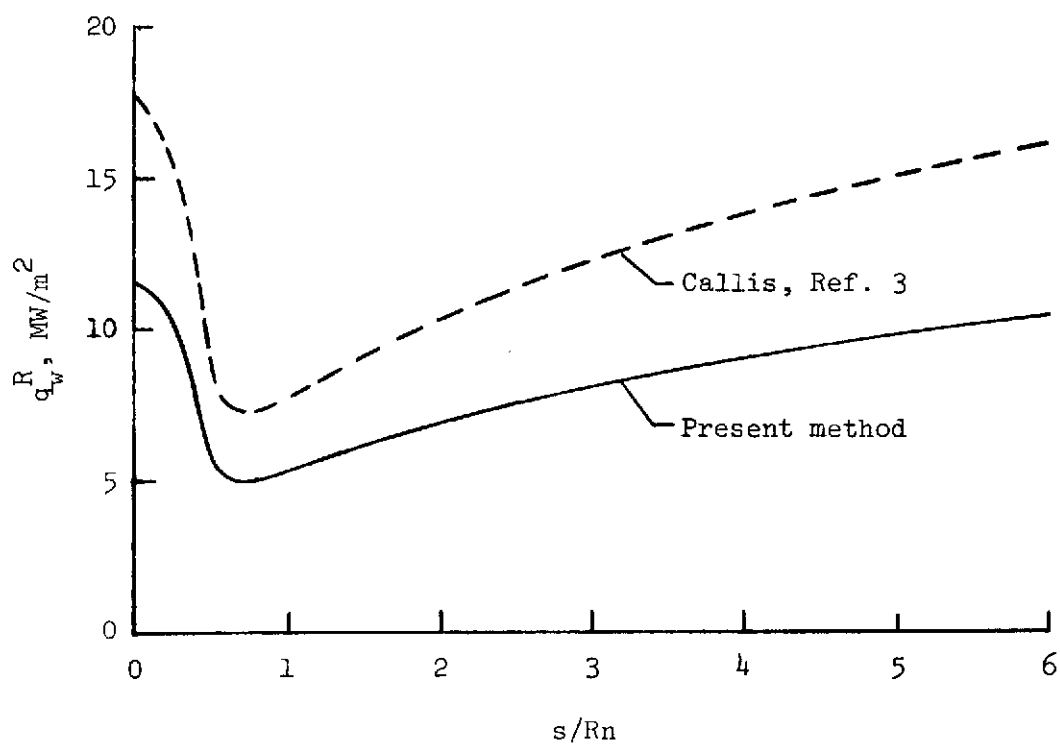


Figure 8. - Radiative heating distribution for free-stream conditions and body shape given in Figure 6(a) except $R_n=2.0$ m. Present results are compared with the method of Callis (ref. 3).

| Case | Gas Composition* | V_∞ , km/s | ρ_∞ , kg/m ³ | p_∞ , N/m ² | h_∞ , MJ/kg | $q_{w,o}^R$, MW/m ² |
|------|--|-------------------|-----------------------------------|-------------------------------|--------------------|---------------------------------|
| A | .79 N ₂ - .21 O ₂ | 12.200 | 2.744×10^{-4} | 20.01 | 0 | 5.78 |
| B | .85 H ₂ - .15 He | 50.000 | 5.700×10^{-3} | 0 | 0 | 9668.00 |
| C | .90 CO ₂ - .10 N ₂ | 8.740 | 3.294×10^{-3} | 130.59 | -8.43 | 4.44 |
| D | .90 CO ₂ - .10 N ₂ | 11.175 | 2.850×10^{-3} | 117.30 | -8.43 | 35.70 |

* by volume

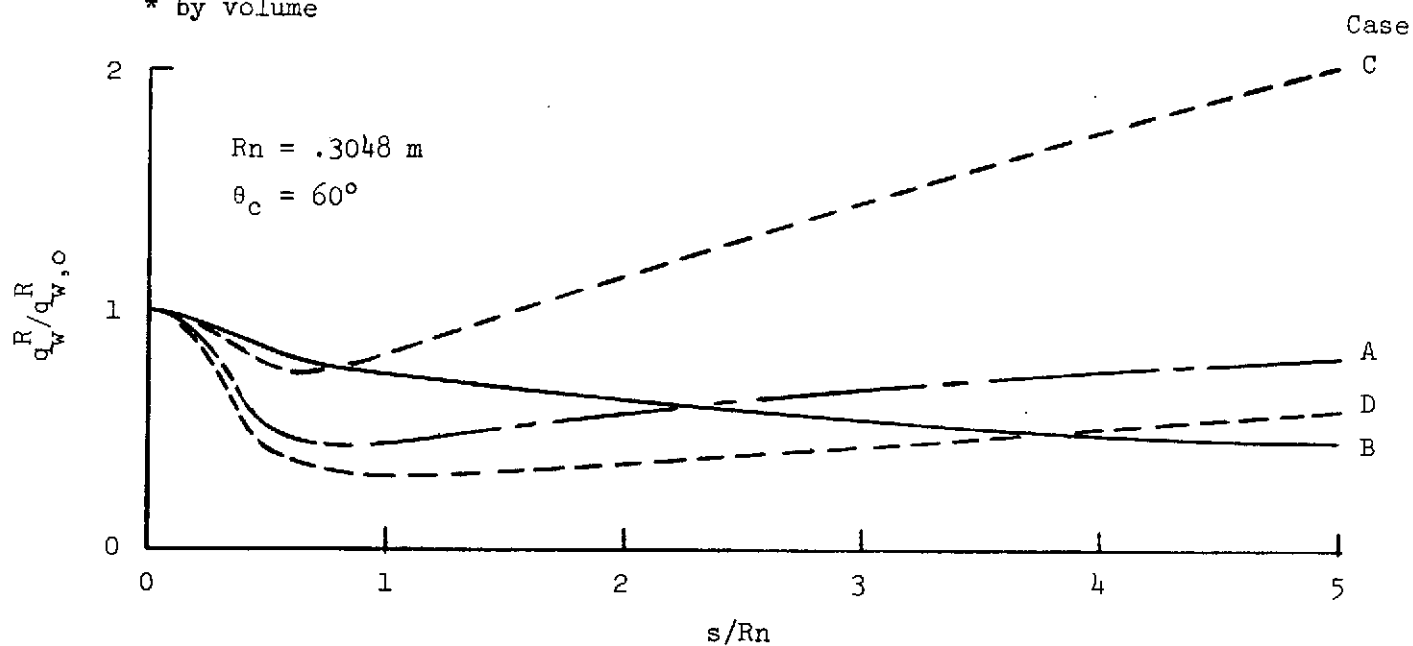


Figure 9. - Radiative heating distributions along a spherically-capped, conical body from radiating, inviscid flow solutions for entries in different atmospheres and different free-stream conditions.

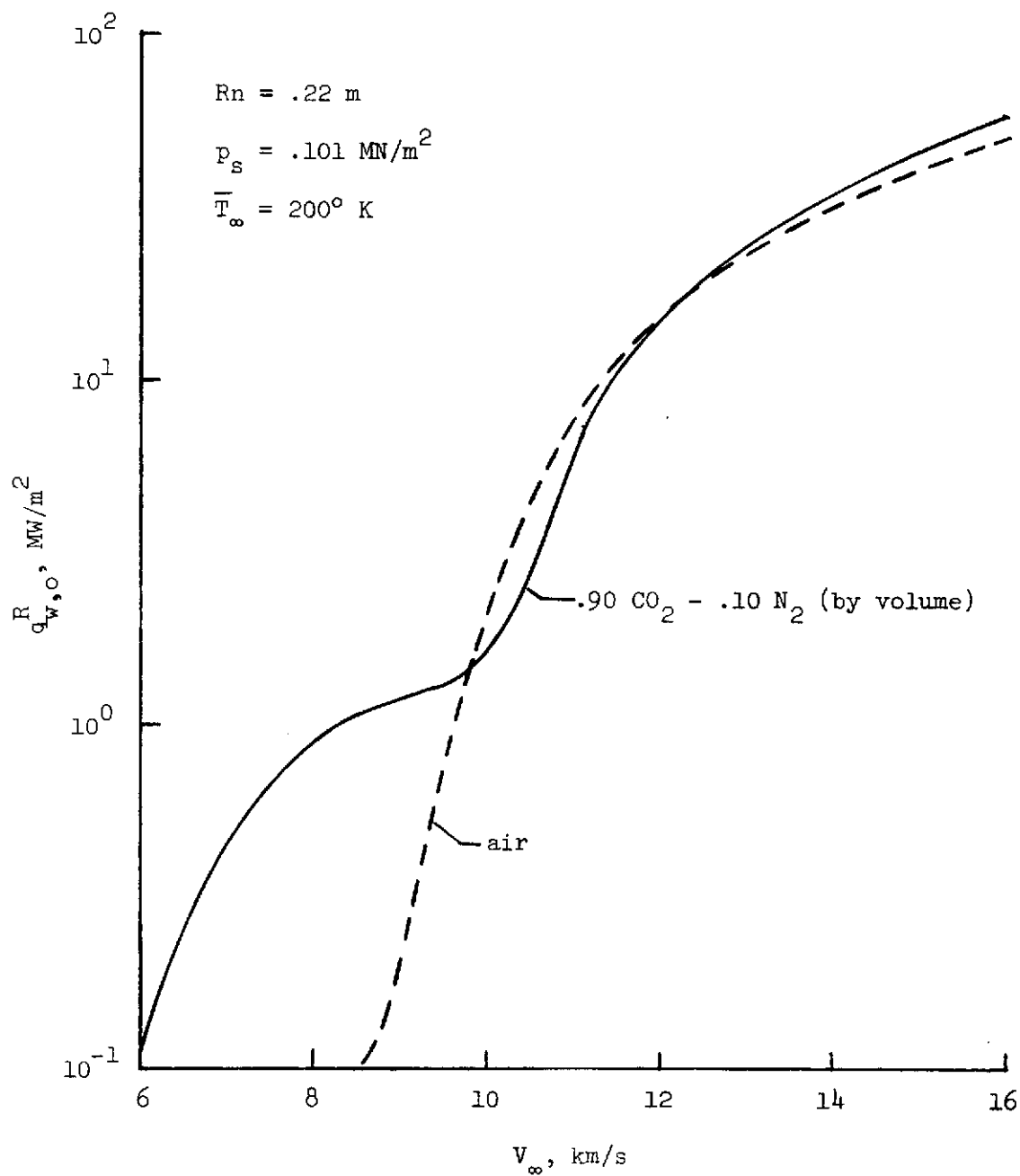


Figure 10. - A comparison of the stagnation-point, radiative heating rates for entries in air and in a .90 CO₂ - .10 N₂ (by volume) gas mixture.

| θ_c | $q_{w,o}^R, \text{ MW/m}^2$ | δ_o/Rn | at $s/Rn=3.0$ | | |
|------------|-----------------------------|---------------|---------------|----------------------------|----------------|
| | | | δ/Rn | \bar{T}_δ/\bar{T}_s | $\theta+\beta$ |
| 60° | 11.39 | .0435 | .1893 | .9058 | 63.33° |
| 55° | 11.35 | .0434 | .1606 | .8514 | 57.77° |
| 50° | 11.33 | .0434 | .1382 | .7644 | 52.23° |
| 45° | 11.33 | .0435 | .1155 | .6276 | 46.59° |

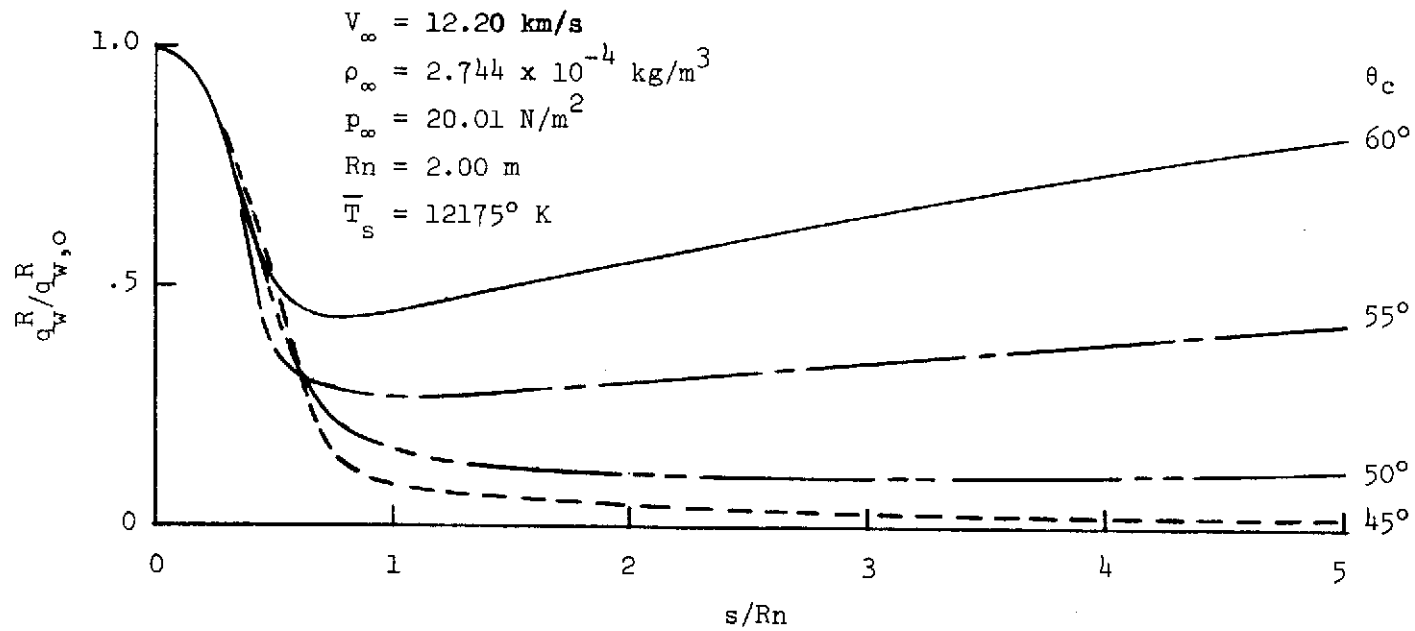
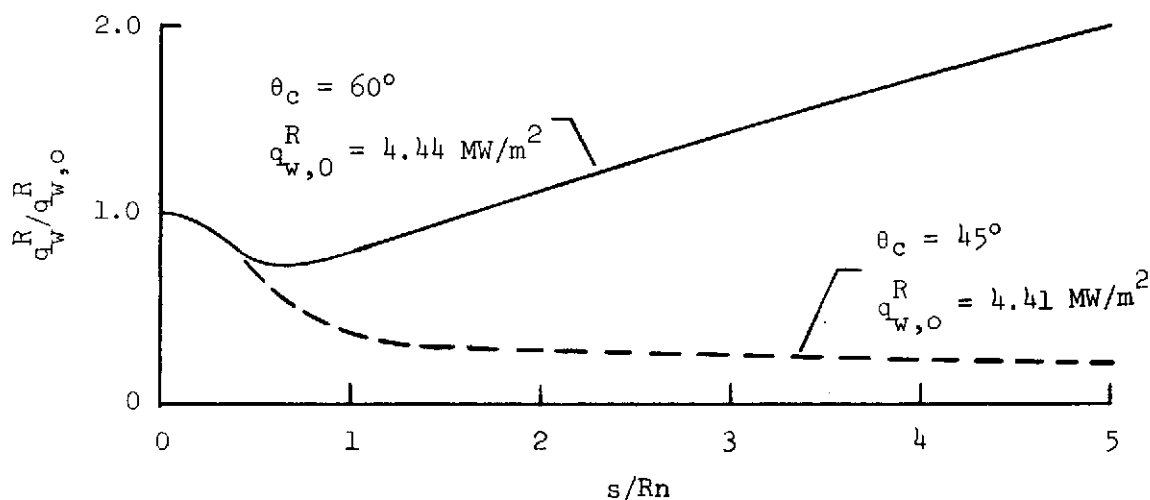
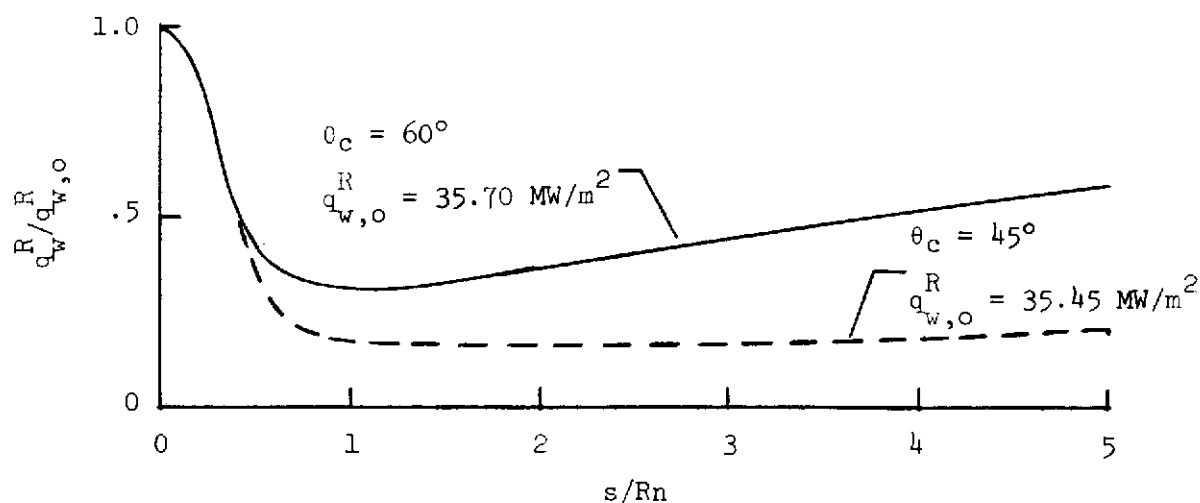


Figure 11. - The distribution of radiative heating along spherically-capped, conical bodies for entry in air.



(a) $V_\infty = 8.740 \text{ km/s}$; $\rho_\infty = 3.294 \times 10^{-3} \text{ kg/m}^3$; $p_\infty = 130.6 \text{ N/m}^2$
 $h_\infty = -8.43 \text{ MJ/kg}$; $Rn = .3048 \text{ m}$



(b) $V_\infty = 11.175 \text{ km/s}$; $\rho_\infty = 2.850 \times 10^{-3} \text{ kg/m}^3$; $p_\infty = 117.3 \text{ N/m}^2$
 $h_\infty = -8.43 \text{ MJ/kg}$; $Rn = .3048 \text{ m}$

Figure 12. - The distribution of radiative heating along spherically-capped, conical bodies for entry in a .90 CO_2 - .10 N_2 (by volume) gas mixture.

$$V_{\infty} = 12.20 \text{ km/s} ; \quad \rho_{\infty} = 2.744 \times 10^{-3} \text{ kg/m}^3$$

$$p_{\infty} = 20.01 \text{ N/m}^2 ; \quad \theta_c = 60^\circ$$

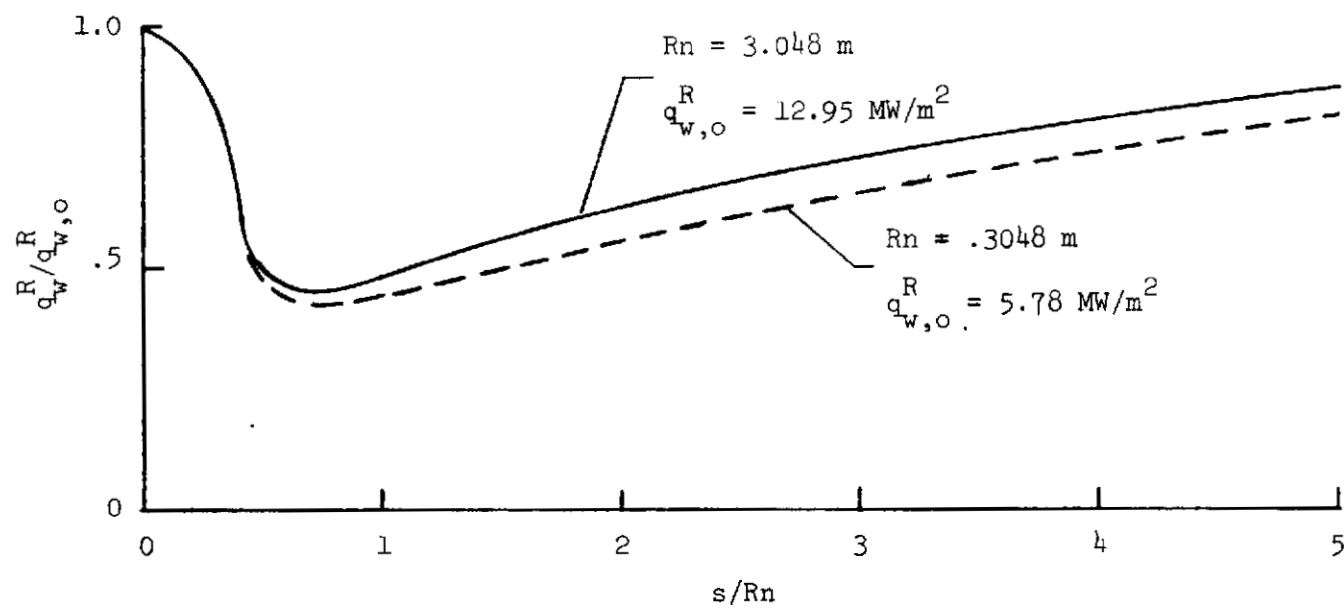
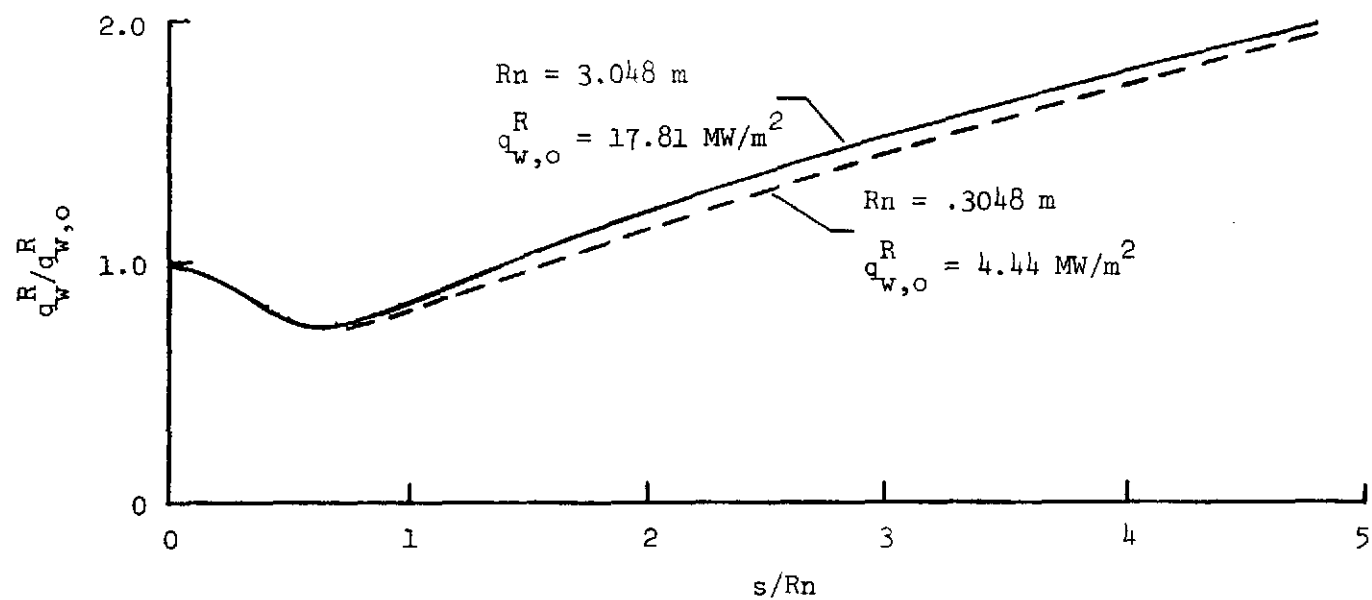
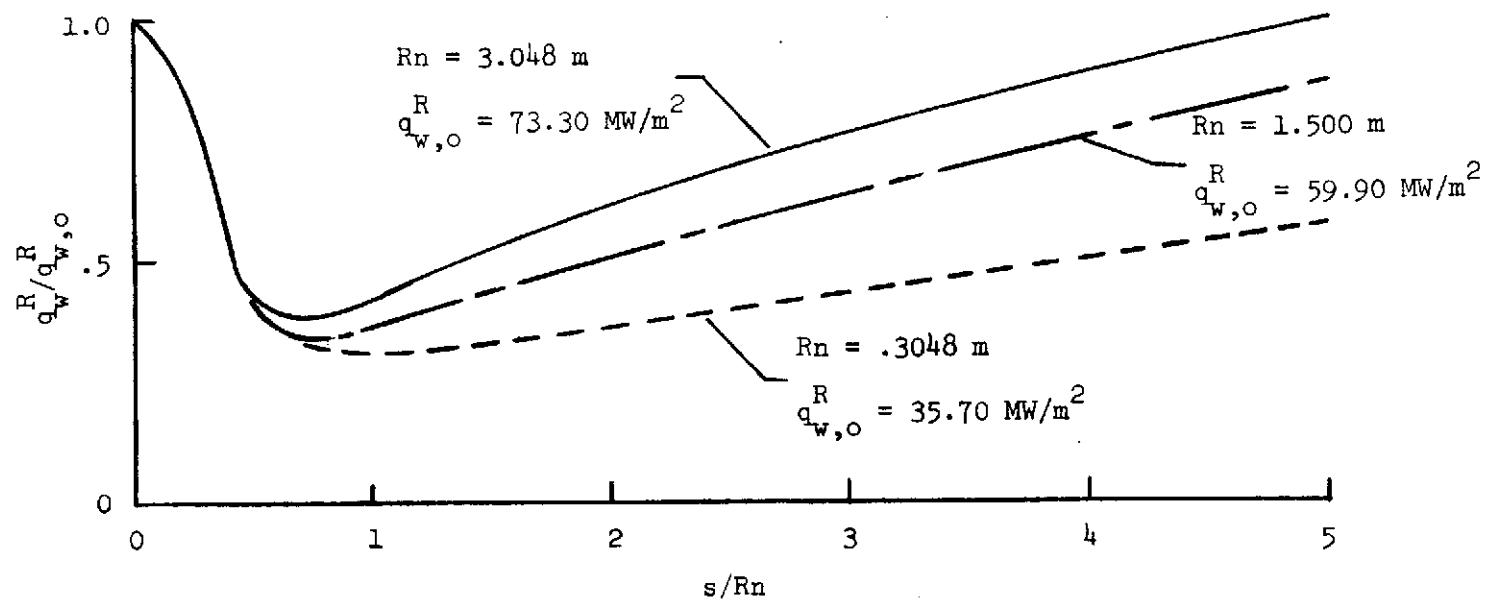


Figure 13. - The effect of nose radius on the distribution of radiative heating along a spherically-capped, conical body for entry in air.



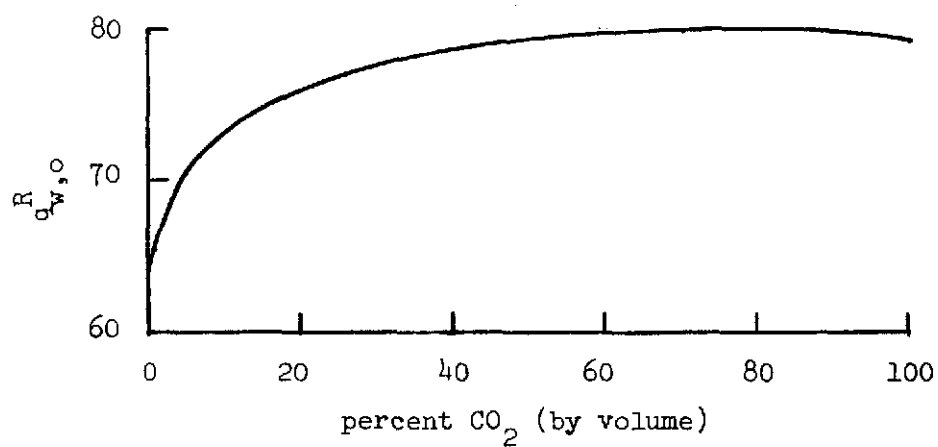
(a) $V_\infty = 8.740 \text{ km/s}$; $\rho_\infty = 3.294 \times 10^{-3} \text{ kg/m}^3$; $p_\infty = 130.6 \text{ N/m}^2$
 $h_\infty = -8.43 \text{ MJ/kg}$; $\theta_c = 60^\circ$

Figure 14. - The effect of nose radius on the distribution of radiative heating along a spherically-capped, conical body for entry in a .90 CO_2 - .10 N_2 (by volume) gas mixture.

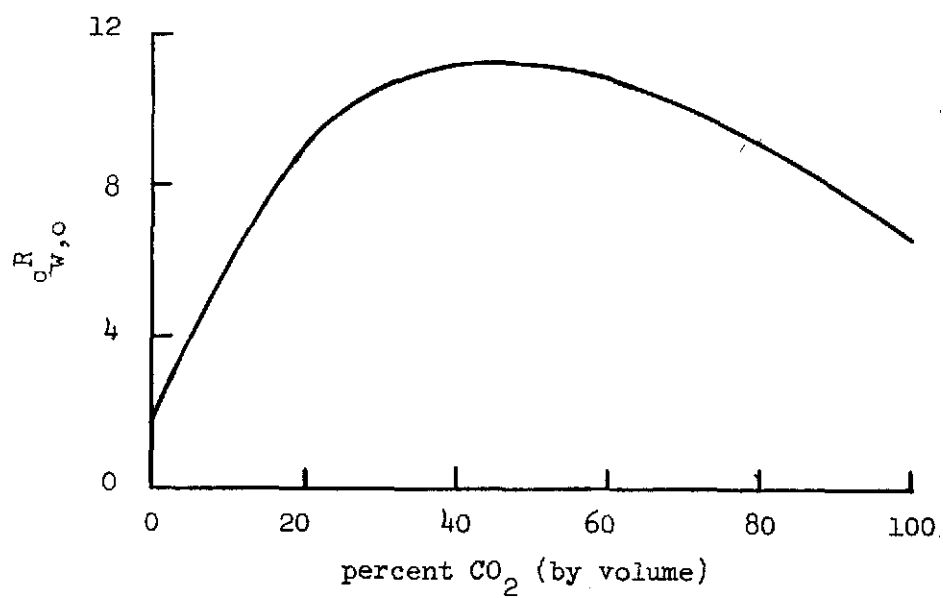


(b) $V_\infty = 11.175 \text{ km/s}$; $\rho_\infty = 2.850 \times 10^{-3} \text{ kg/m}^3$; $p_\infty = 117.3 \text{ N/m}^2$
 $h_\infty = -8.43 \text{ MJ/kg}$; $\theta_c = 60^\circ$

Figure 14. - Concluded.



(a) $V_\infty = 12.0$ km/s ; $\rho_\infty = 2.99 \times 10^{-3}$ kg/m³



(b) $V_\infty = 9.0$ km/s ; $\rho_\infty = 5.31 \times 10^{-3}$ kg/m³

Figure 15. - Stagnation-point, radiative heating rates in CO_2 - N_2 gas mixtures. $Rn = .3048$ m

Gas composition by volume

.79 N₂ - .21 O₂ (air)

$V_{\infty} = 15.250 \text{ km/s}$

$\rho_{\infty} = 1.77 \times 10^{-4} \text{ kg/m}^3$

$p_{\infty} = 12.3 \text{ N/m}^2$

$h_{\infty} = 0.0 \text{ J/kg}$

$Rn = 2.56 \text{ m}$

Specified wall conditions

$\bar{T}_w = 3840 \text{ }^{\circ}\text{K}$

$(\rho v)_w = .205 \text{ kg/m}^2\text{-s}$

Elemental mass fractions of
injected gas:

Carbon .923

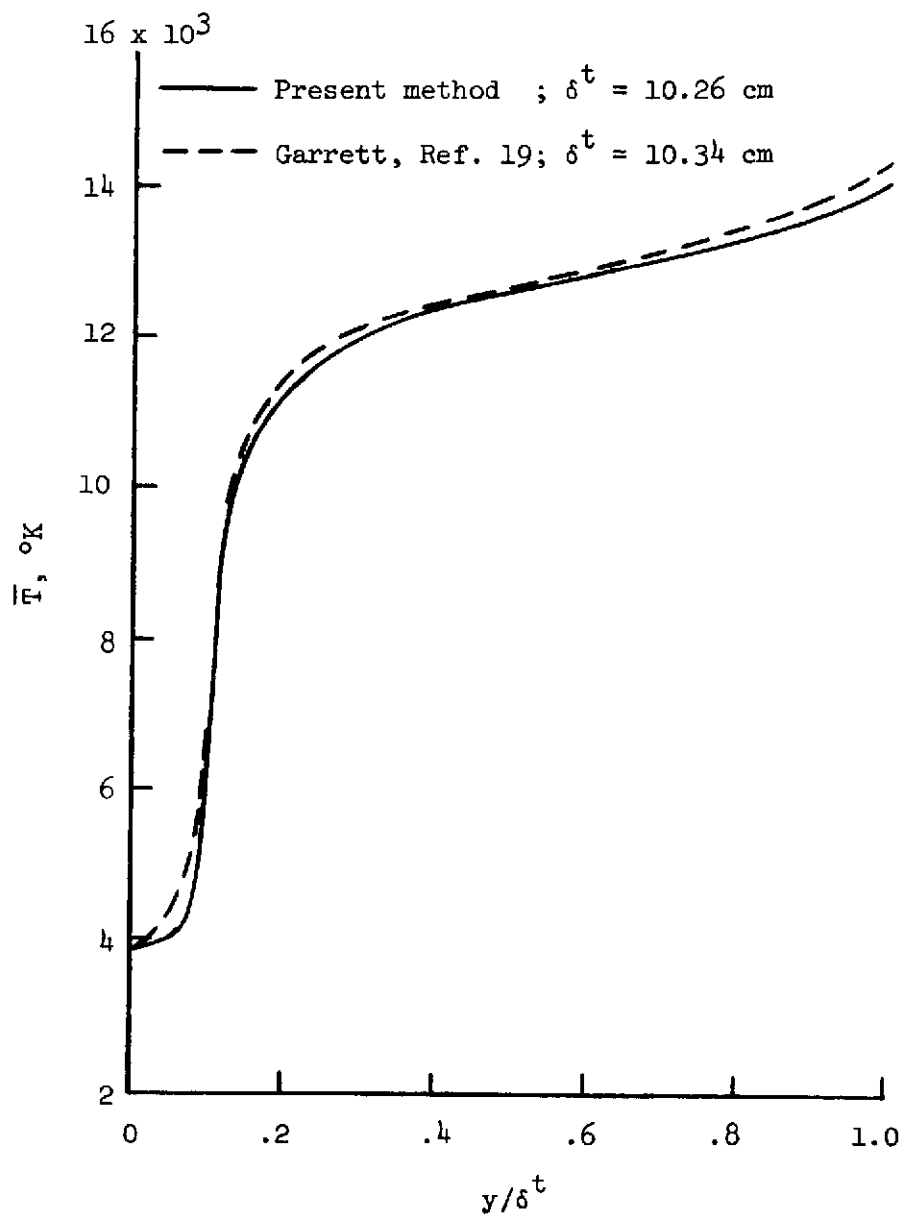
Oxygen .058

Hydrogen .018

Nitrogen .001

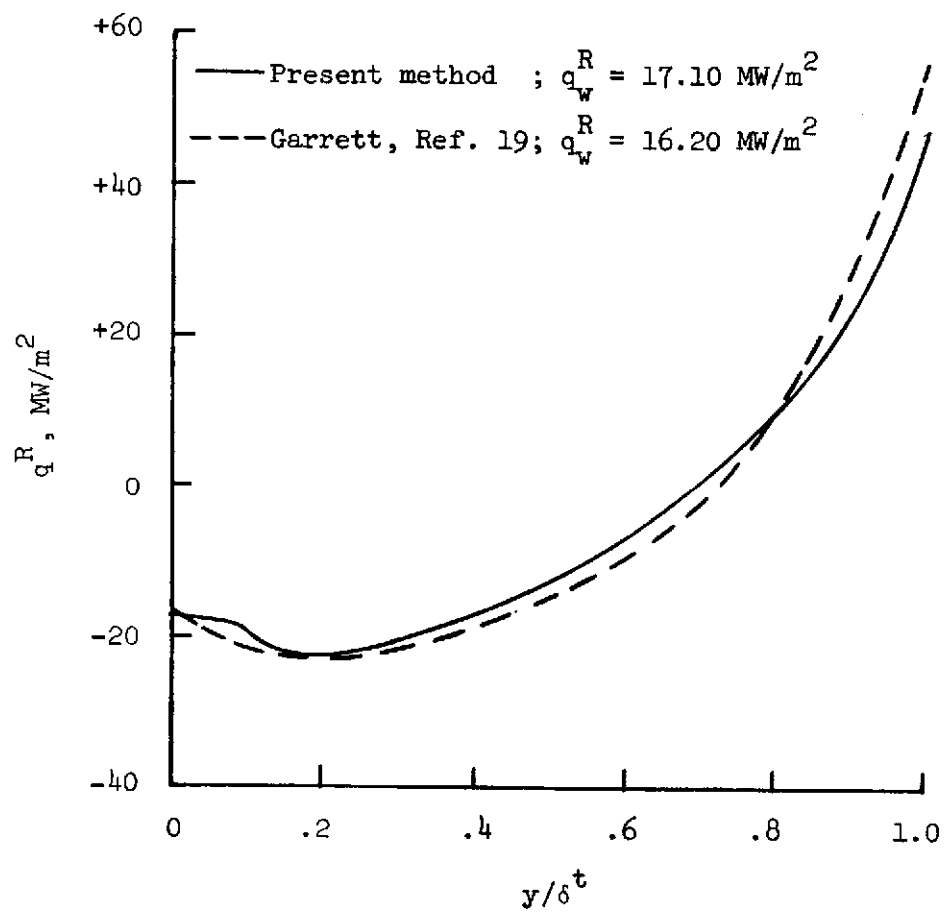
(a) Free-stream conditions and specified wall conditions.

Figure 16. - Fully-coupled, stagnation-point solution for Earth reentry. Present method is compared with the method of Garrett (ref. 19).



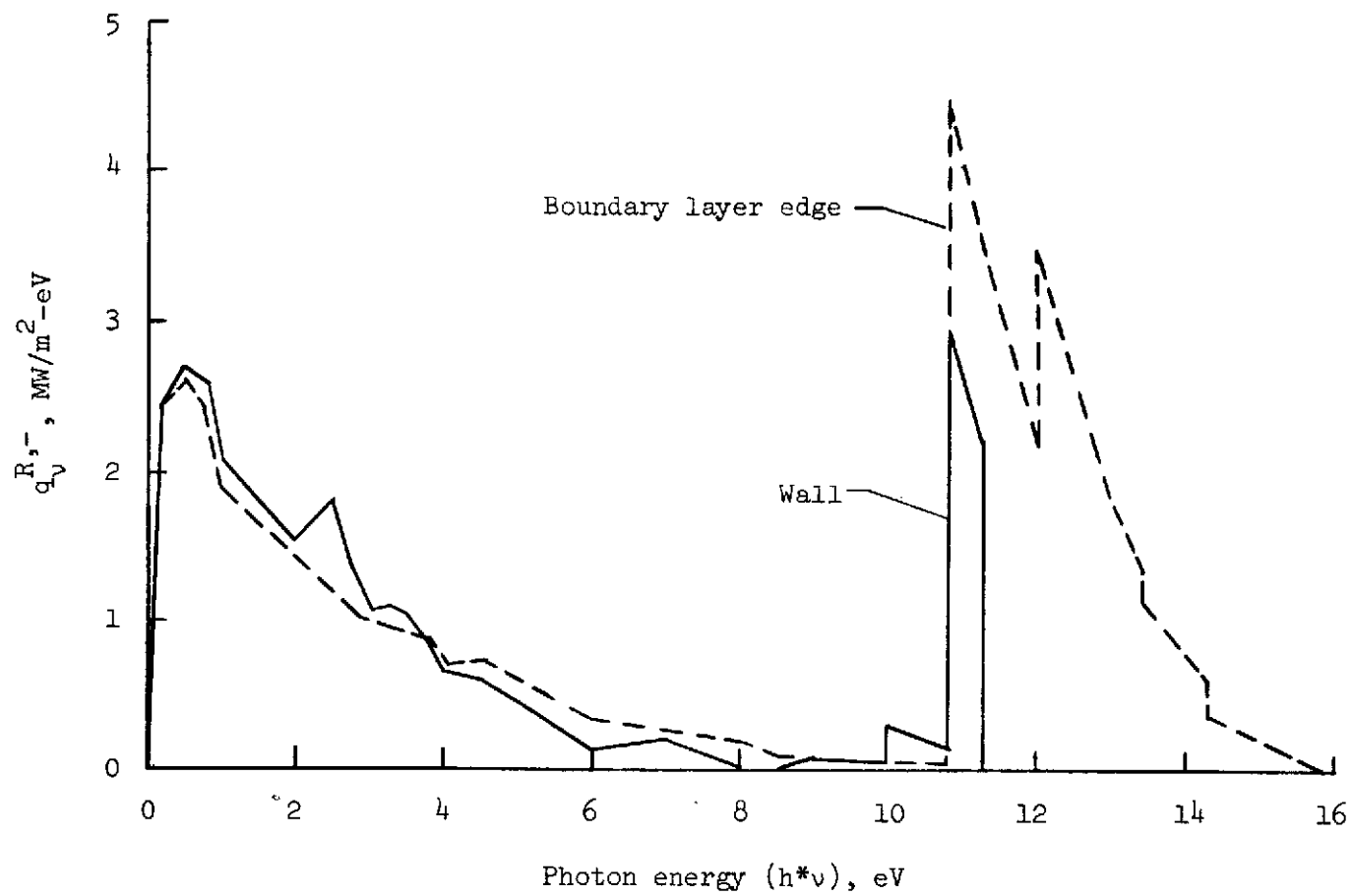
(b) Temperature profile through layer.

Figure 16. - Continued.



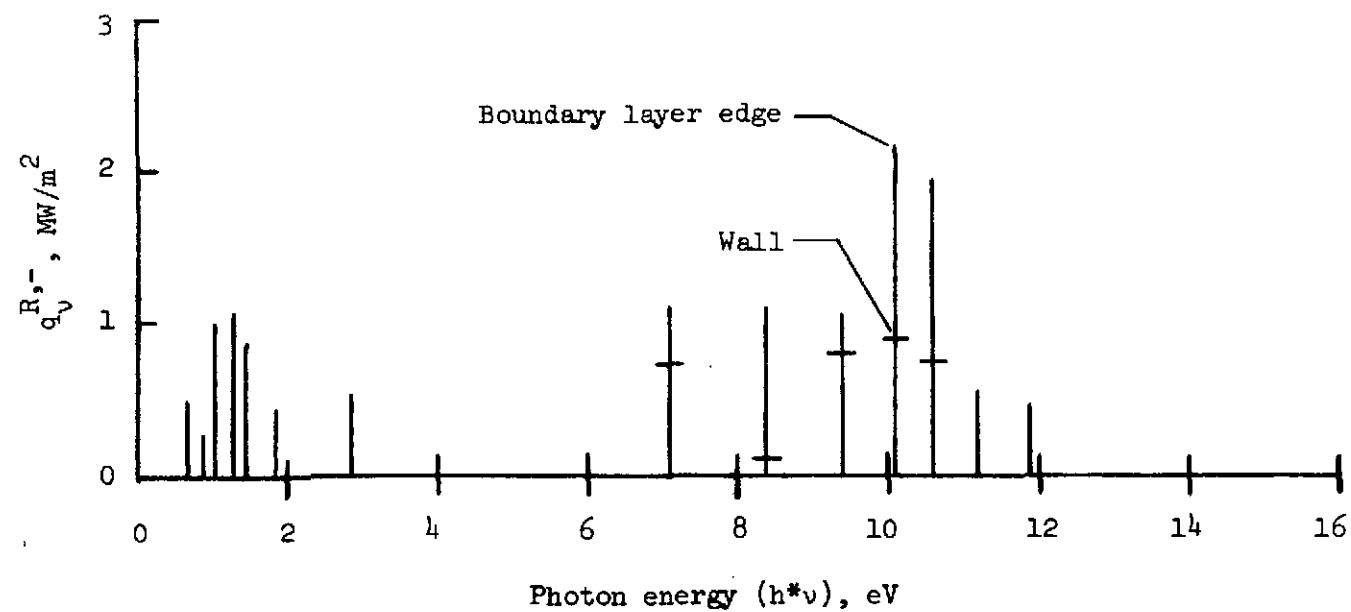
(c) Radiative heating profile through layer.

Figure 16. - Continued.



(d) Spectral distribution of radiative heating toward the body due to continuum processes.

Figure 16. - Continued.



(e) Spectral distribution of radiative heating toward the body due to line processes.

Figure 16. - Concluded.

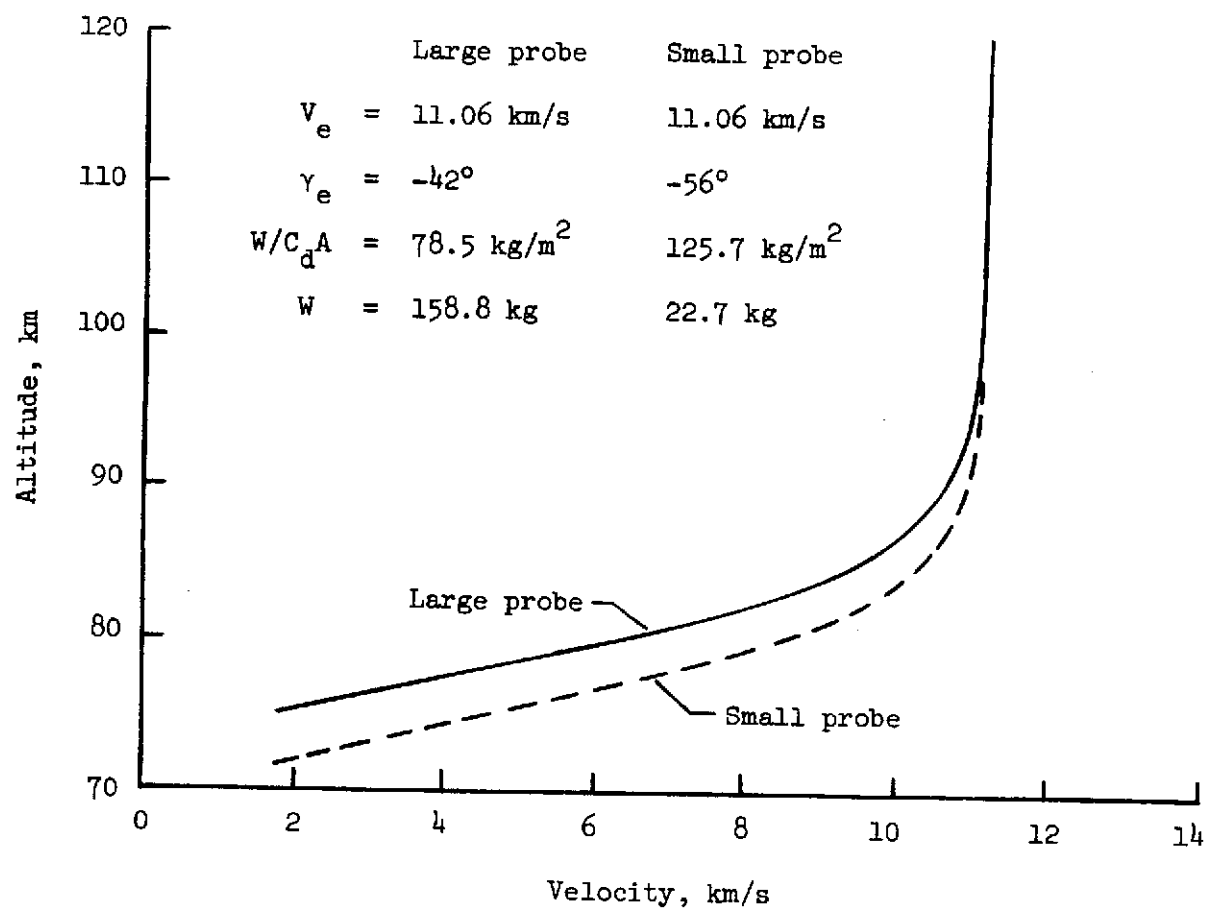


Figure 17. - Trajectories for large and small probes during entry to Venus.

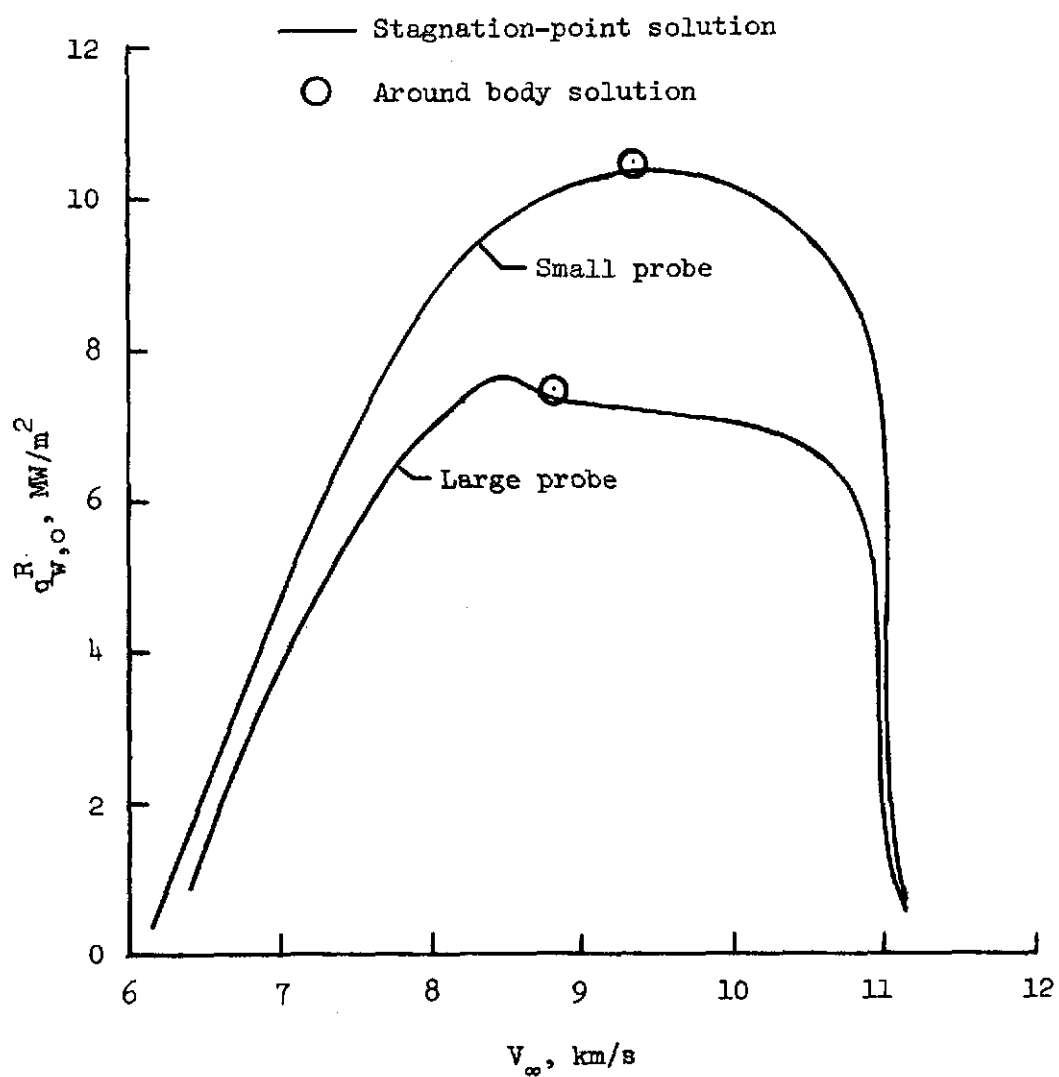


Figure 18. - Stagnation-point, radiative heating rates along entry trajectories for large and small probes. Results from radiating, inviscid flow solutions.

Gas composition by volume
 $.97 \text{ CO}_2 - .03 \text{ N}_2$

$$V_\infty = 8.803 \text{ km/s}$$

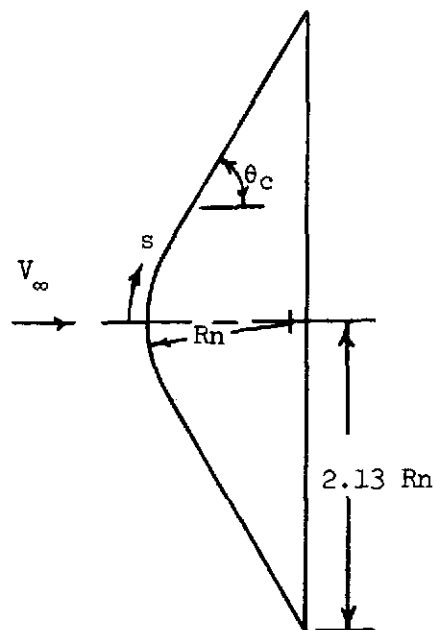
$$\rho_\infty = 5.79 \times 10^{-3} \text{ kg/m}^3$$

$$p_\infty = 200.1 \text{ N/m}^2$$

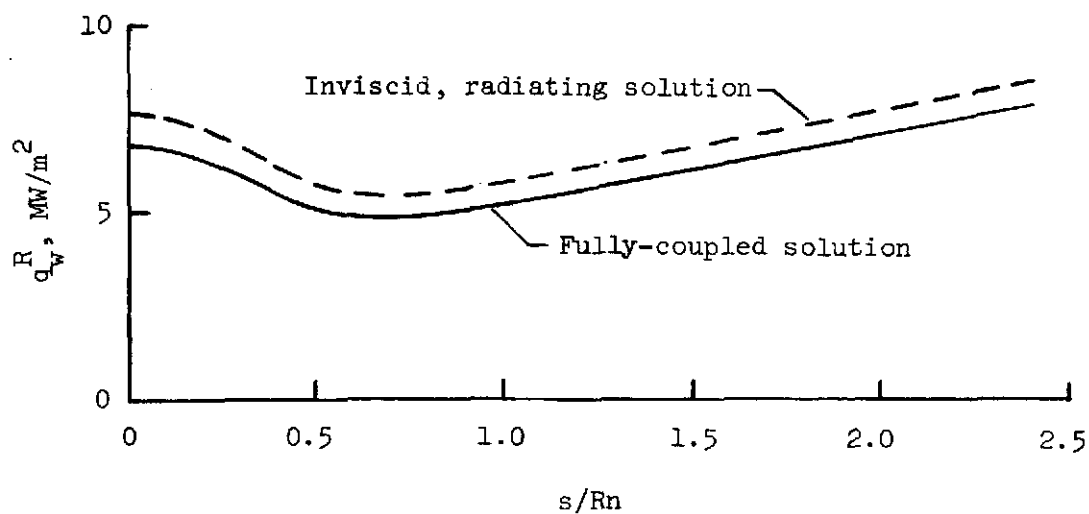
$$h_\infty = -8.86 \text{ MJ/kg}$$

$$R_n = 0.3429 \text{ m}$$

$$\theta_c = 60^\circ$$

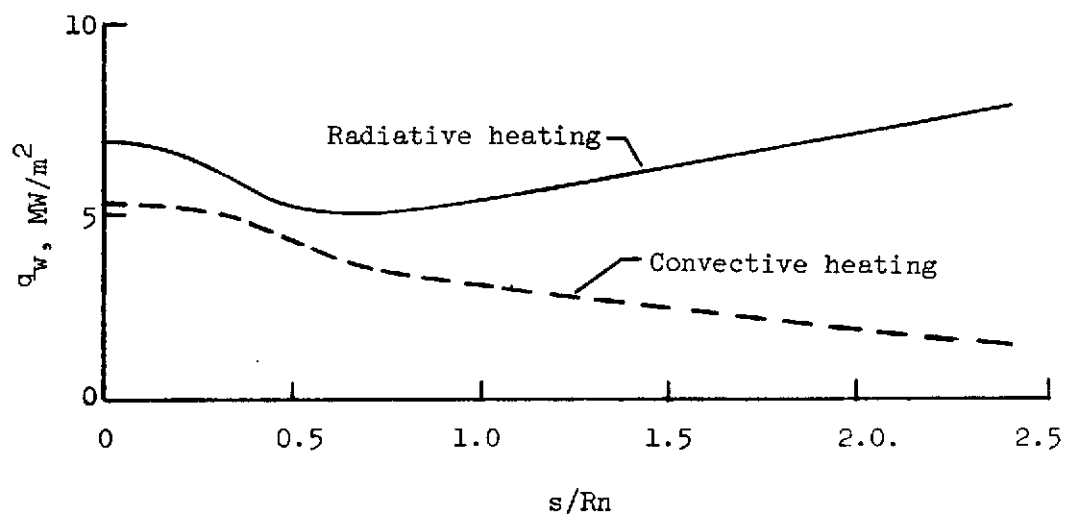


(a) Free-stream conditions and body shape.

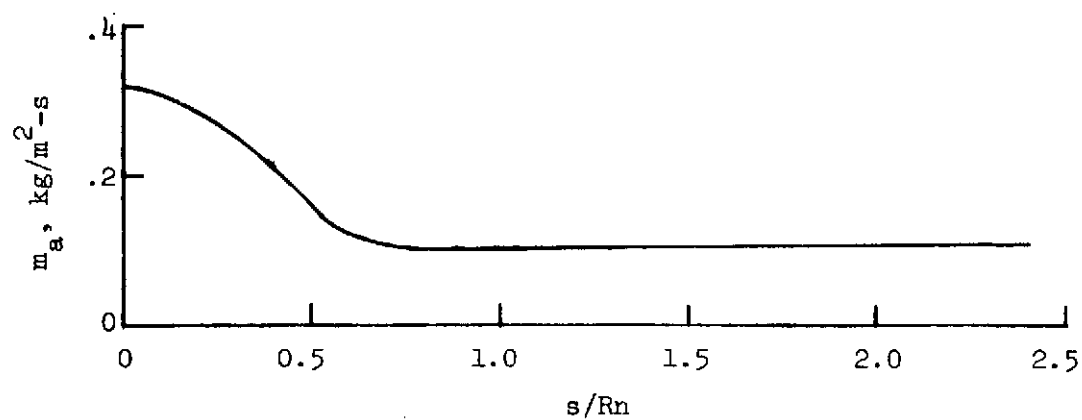


(b) Comparison of radiative heating.

Figure 19. - Fully-coupled, radiating flow solution with steady-state ablation of carbon-phenolic heatshield for large probe.

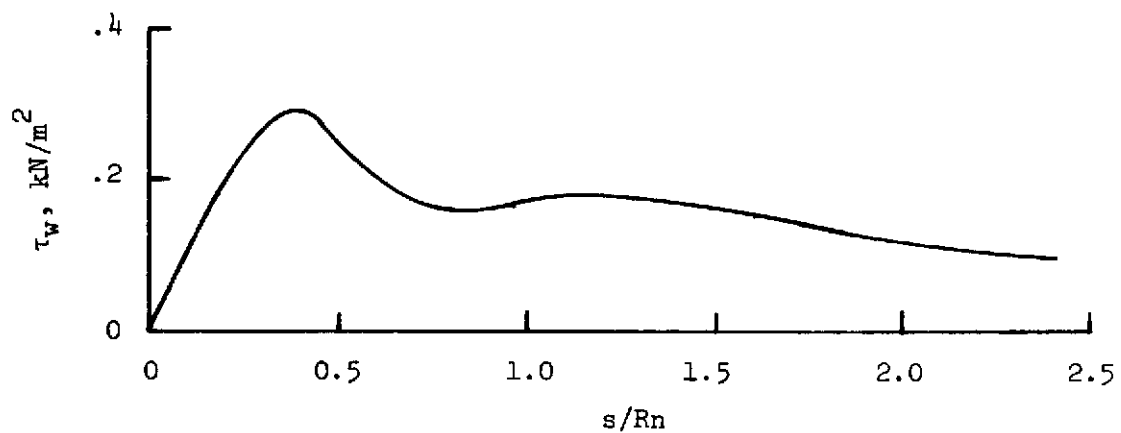


(c) Radiative and convective heating distribution.

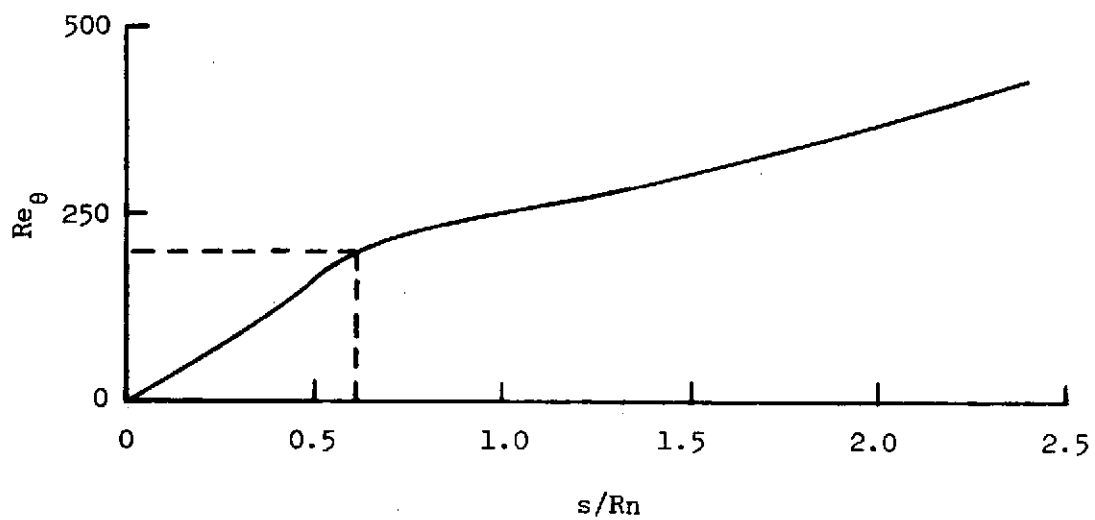


(d) Ablation rate distribution.

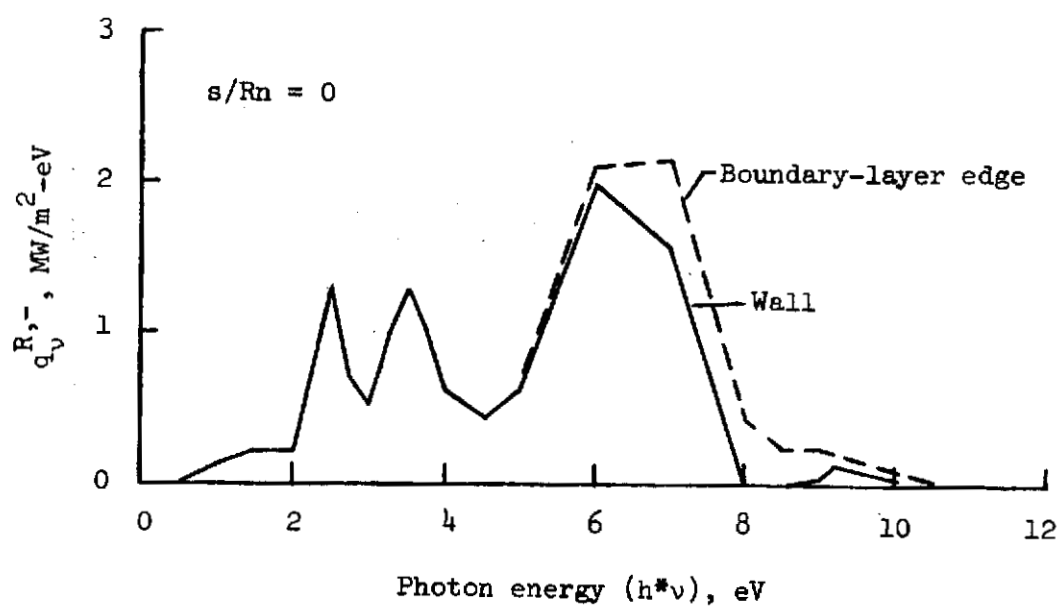
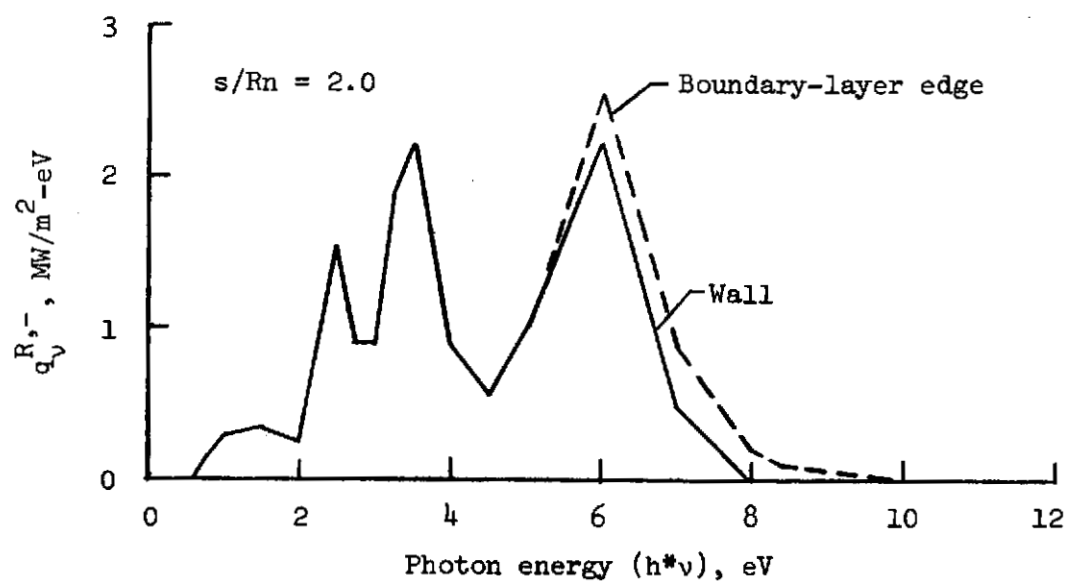
Figure 19. - Continued.



(e) Aerodynamic shear distribution.



(f) Momentum thickness Reynolds number distribution.



(g) Spectral distribution of radiative heating toward the body due to continuum processes.

Gas composition by volume
 .97 CO₂ - .03 N₂

$V_{\infty} = 9.331 \text{ km/s}$

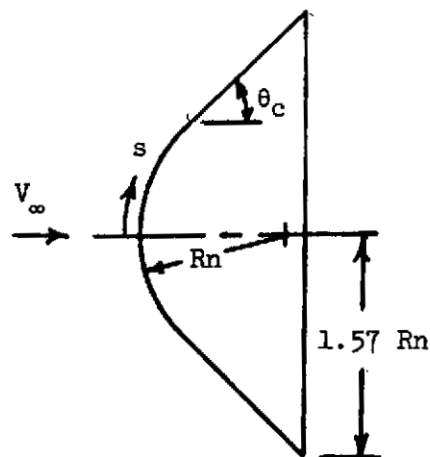
$\rho_{\infty} = 9.32 \times 10^{-3} \text{ kg/m}^3$

$p_{\infty} = 330.4 \text{ N/m}^2$

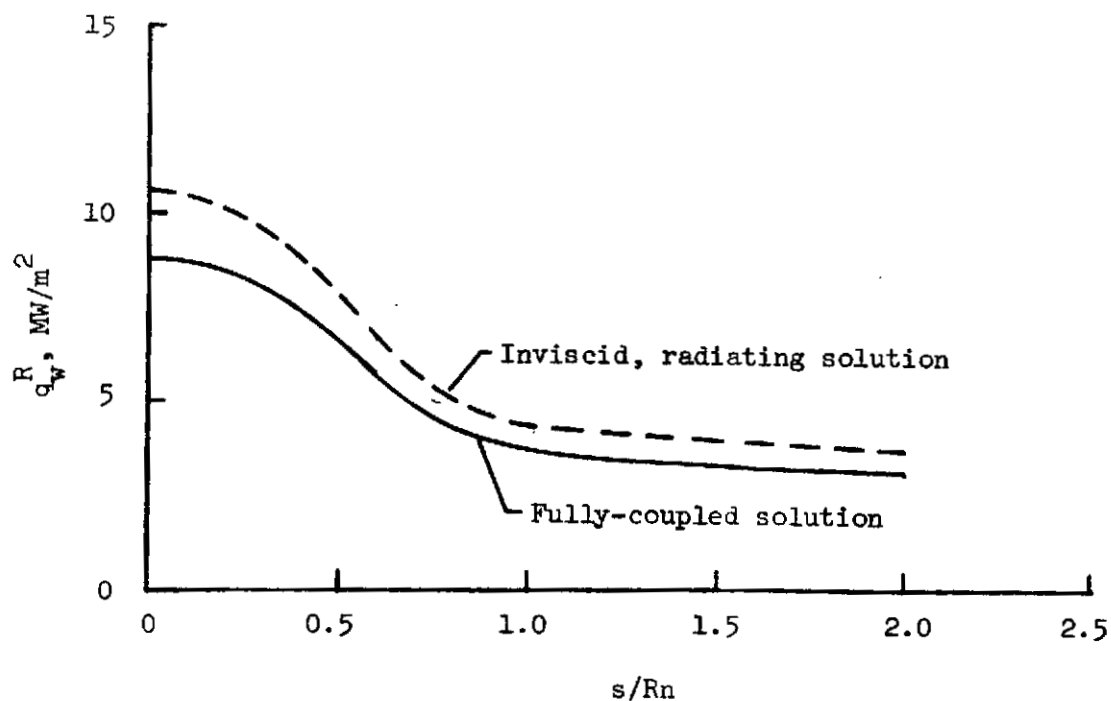
$h_{\infty} = -8.86 \text{ MJ/kg}$

$R_n = 0.1397 \text{ m}$

$\theta_c = 45^\circ$

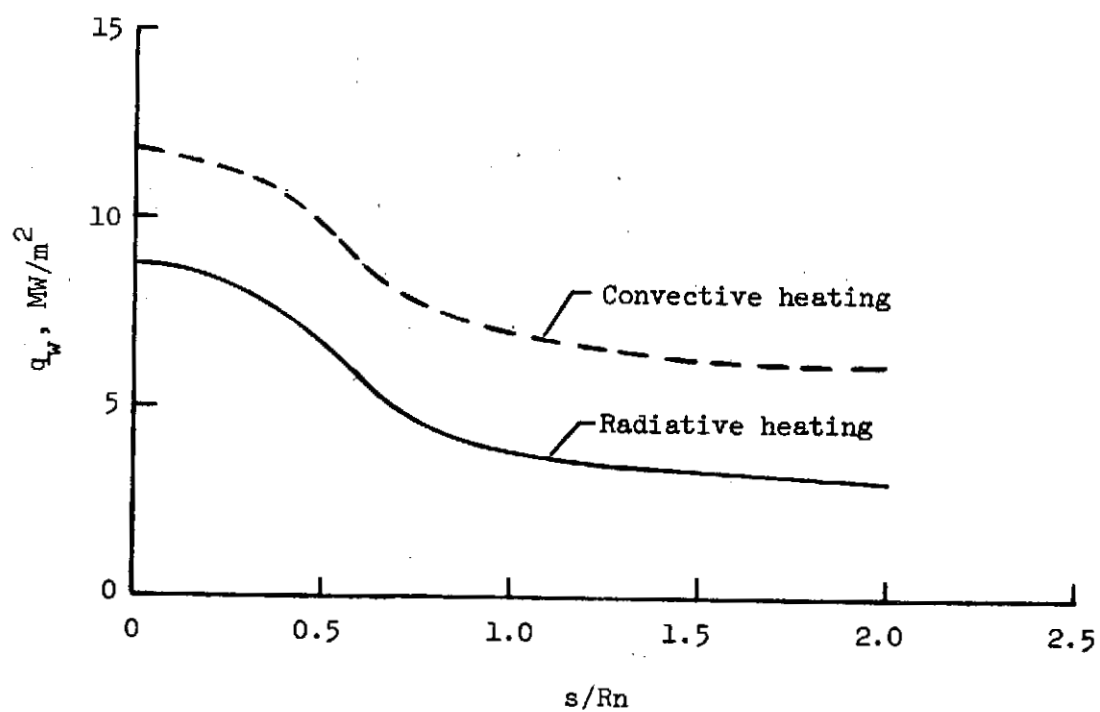


(a) Free-stream conditions and body shape.

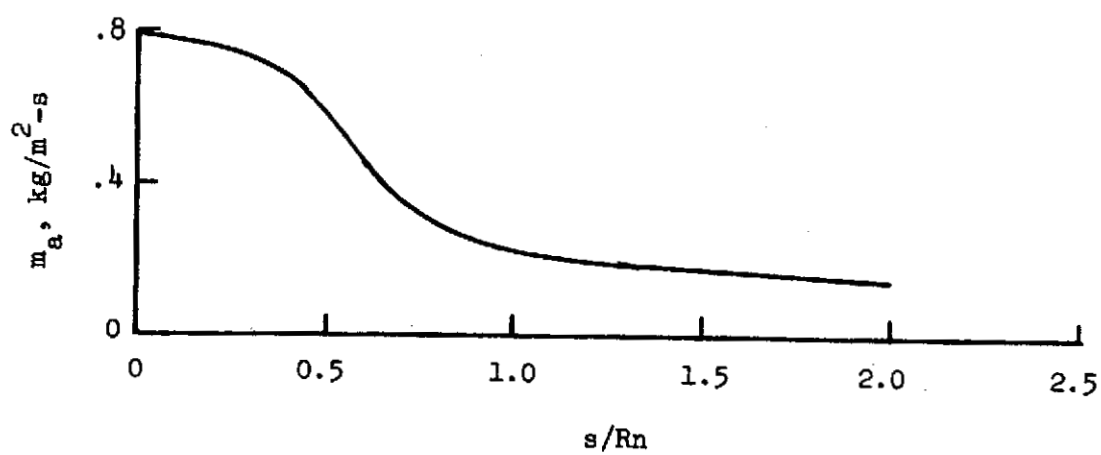


(b) Comparison of radiative heating.

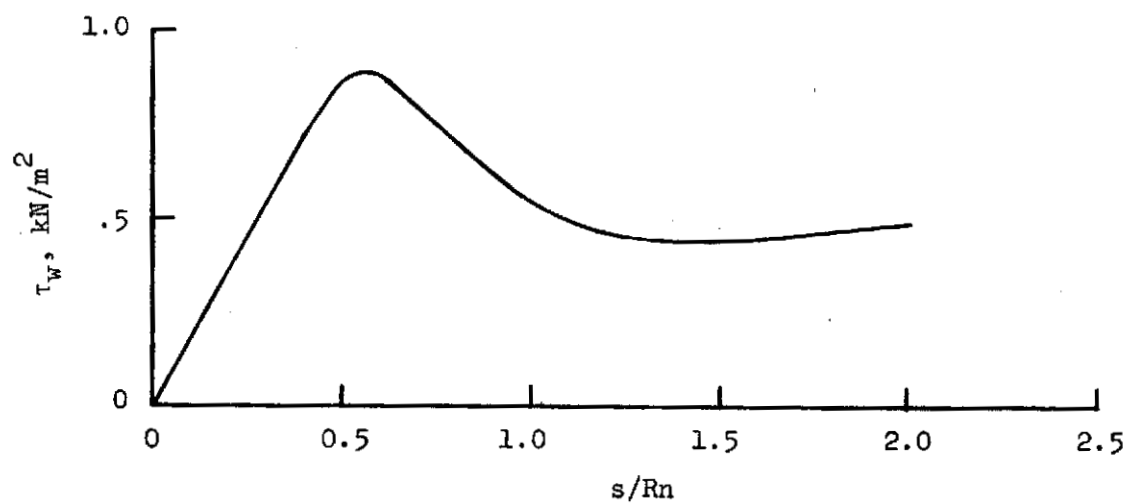
Figure 20. - Fully-coupled, radiating flow solution with steady-state ablation of carbon-phenolic heatshield for small probe.



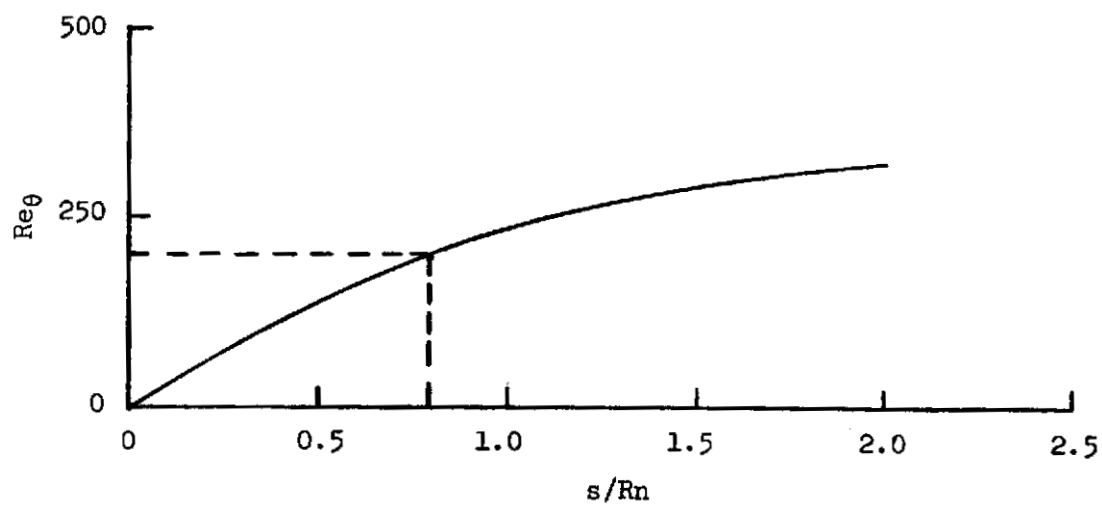
(c) Radiative and convective heating distribution.



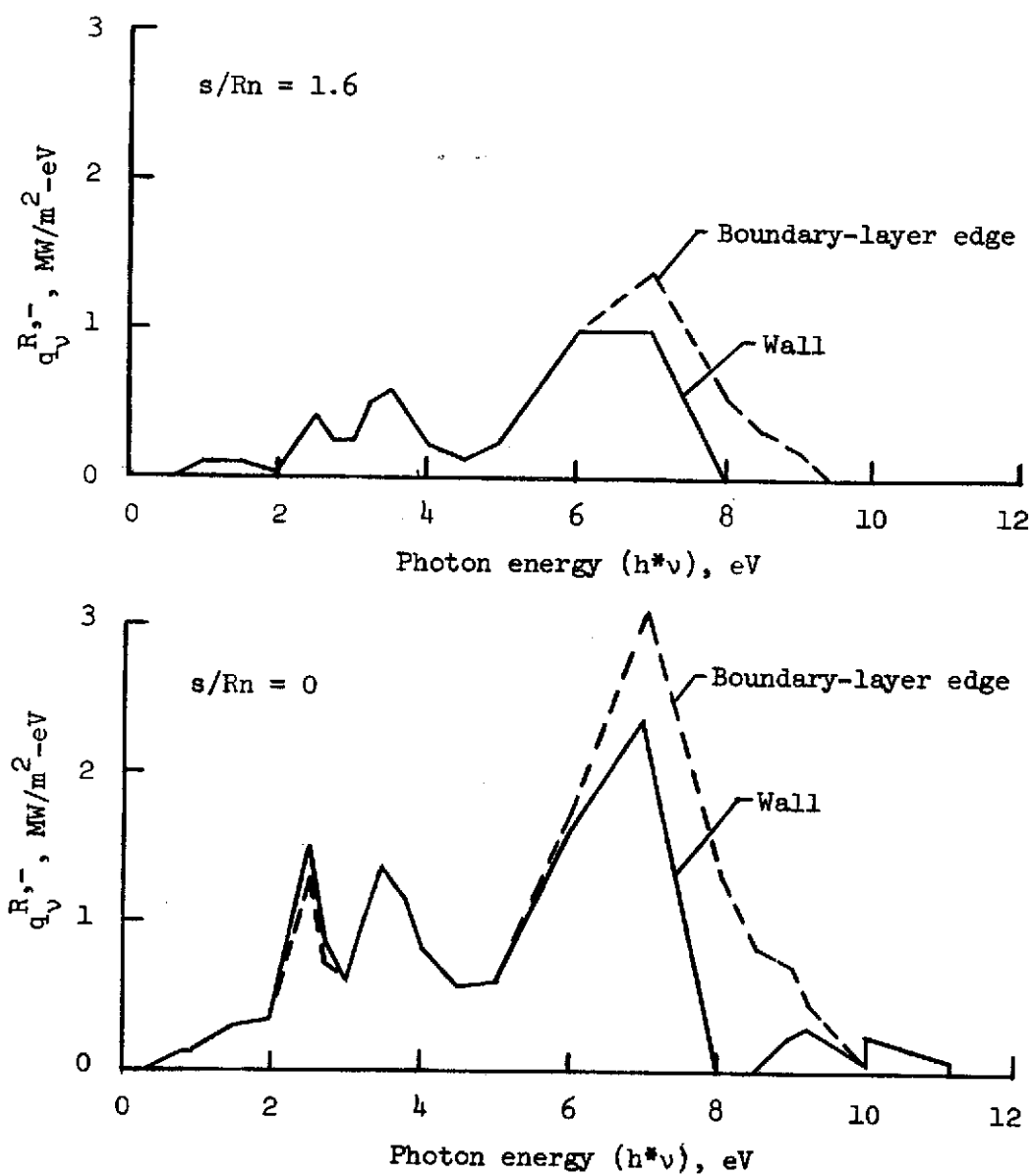
(d) Ablation rate distribution.



(e) Aerodynamic shear distribution.

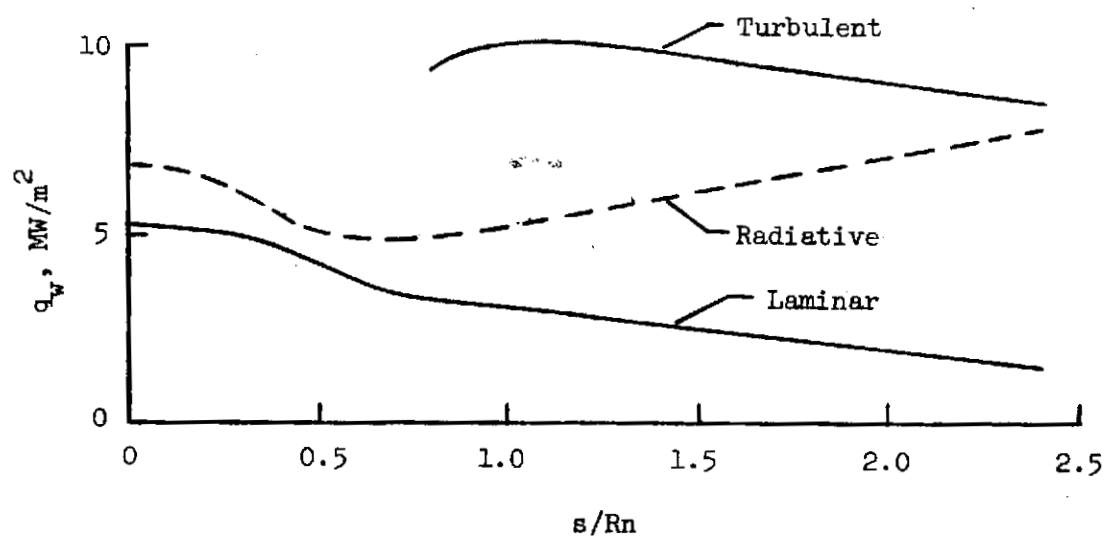


(f) Momentum thickness Reynolds number distribution.

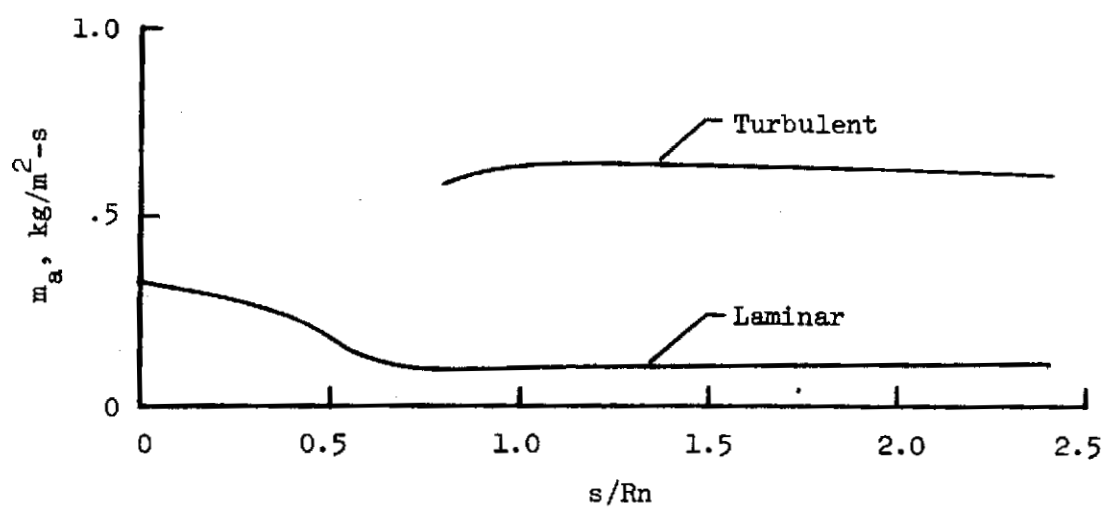


(g) Spectral distribution of radiative heating toward the body due to continuum processes.

Figure 20.- Concluded.

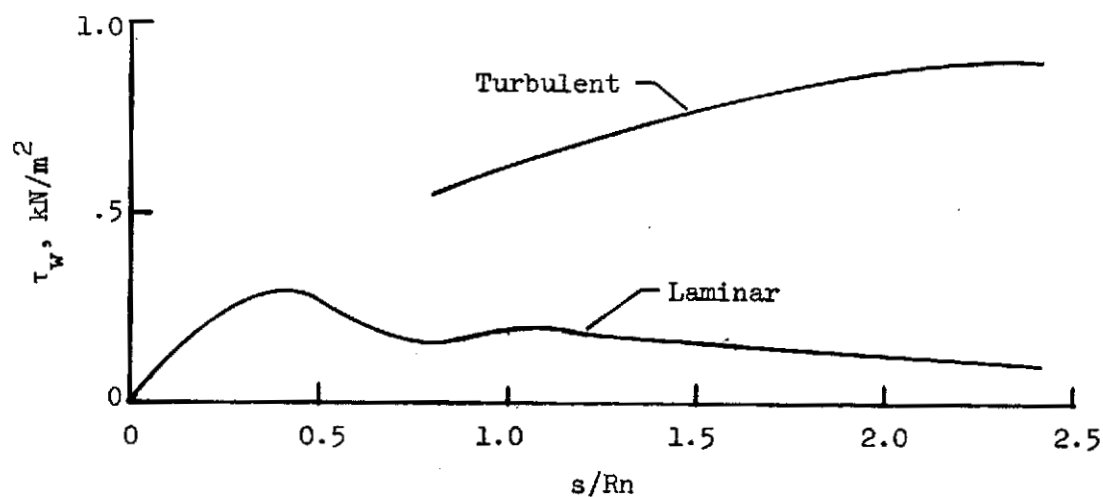


(a) Heating rate distribution.

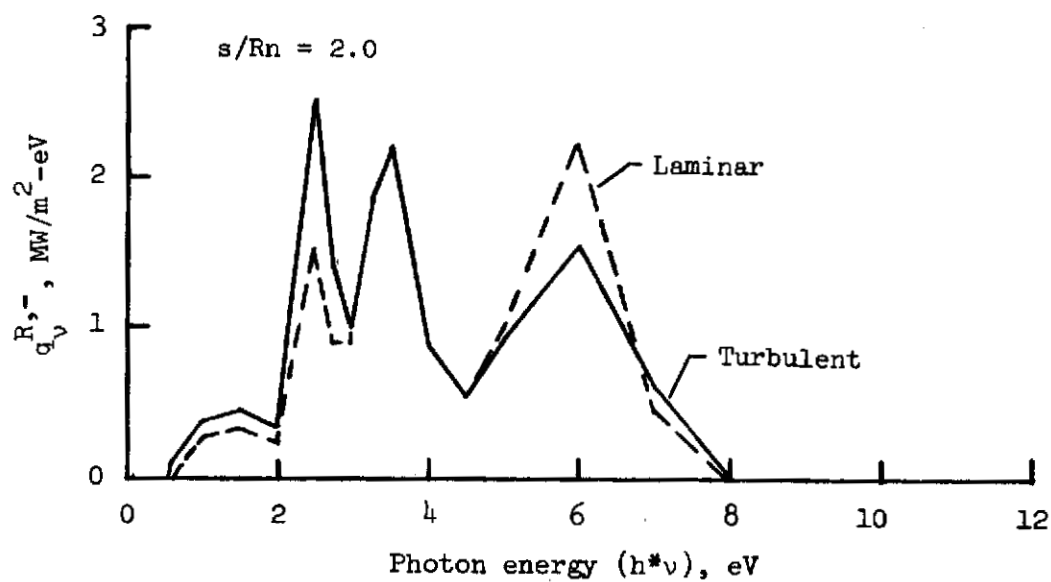


(b) Ablation rate distribution.

Figure 21. - Comparison between a turbulent and laminar boundary layer for fully-coupled solution for large probe.



(c) Aerodynamic shear distribution.



(d) Spectral distribution at the wall of radiative heating toward the body due to continuum processes.

Gas composition by volume
 .97 CO₂ - .03 N₂

$$V_{\infty} = 11.175 \text{ km/s}$$

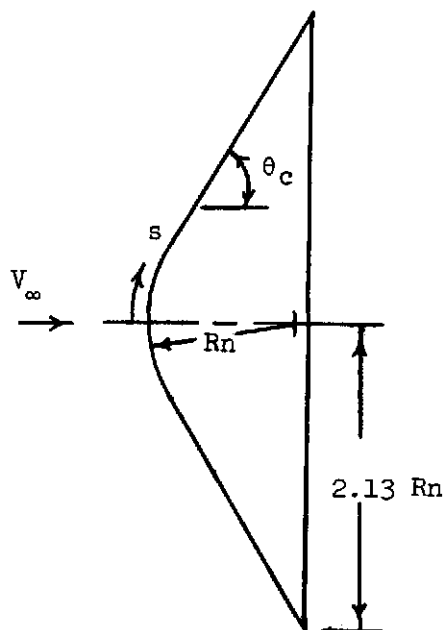
$$\rho_{\infty} = 2.85 \times 10^{-3} \text{ kg/m}^3$$

$$p_{\infty} = 117.3 \text{ N/m}^2$$

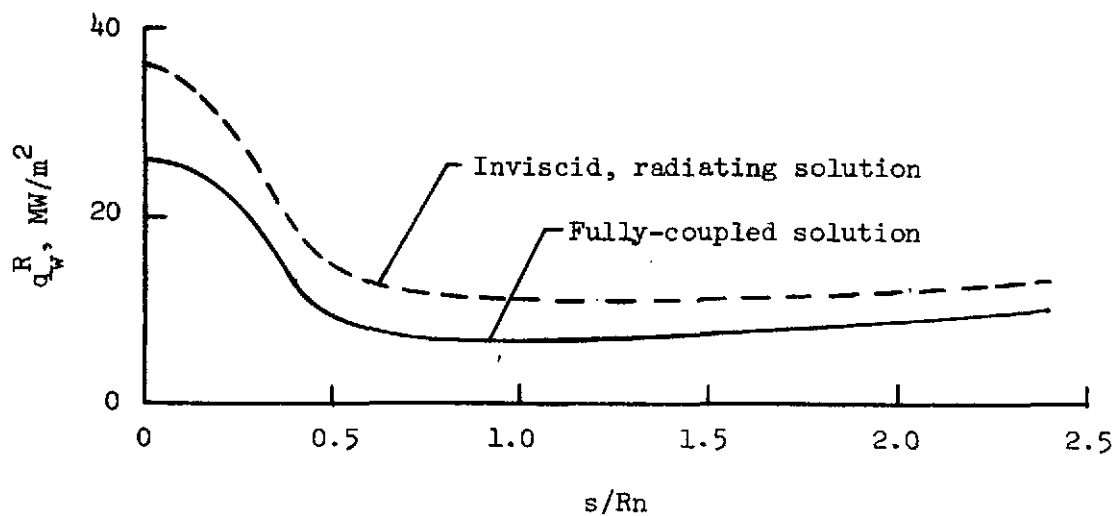
$$h_{\infty} = -8.86 \text{ MJ/kg}$$

$$Rn = 0.3048 \text{ m}$$

$$\theta_c = 60^\circ$$

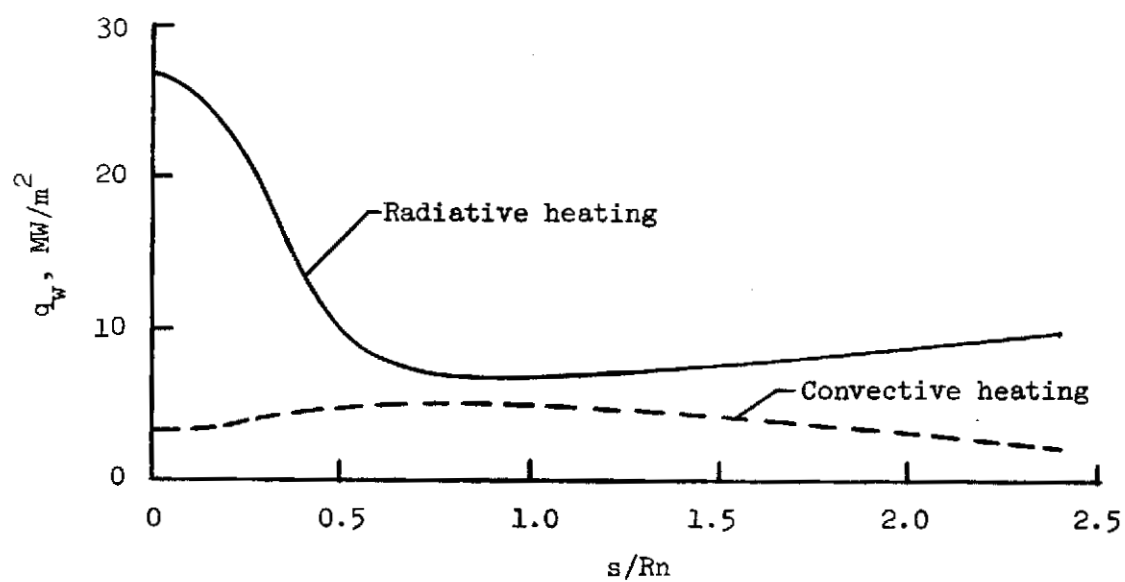


(a) Free-stream conditions and body shape.

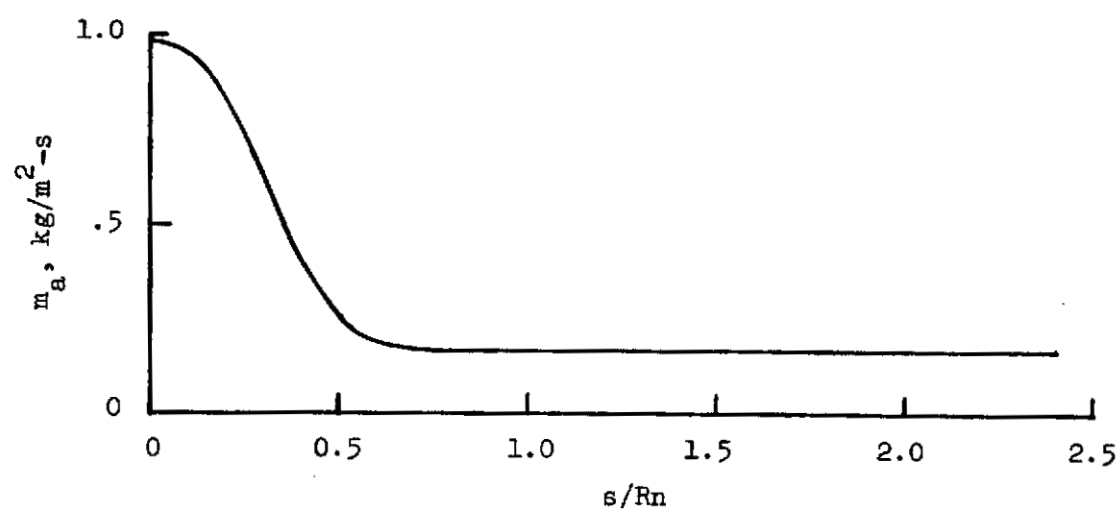


(b) Comparison of radiative heating.

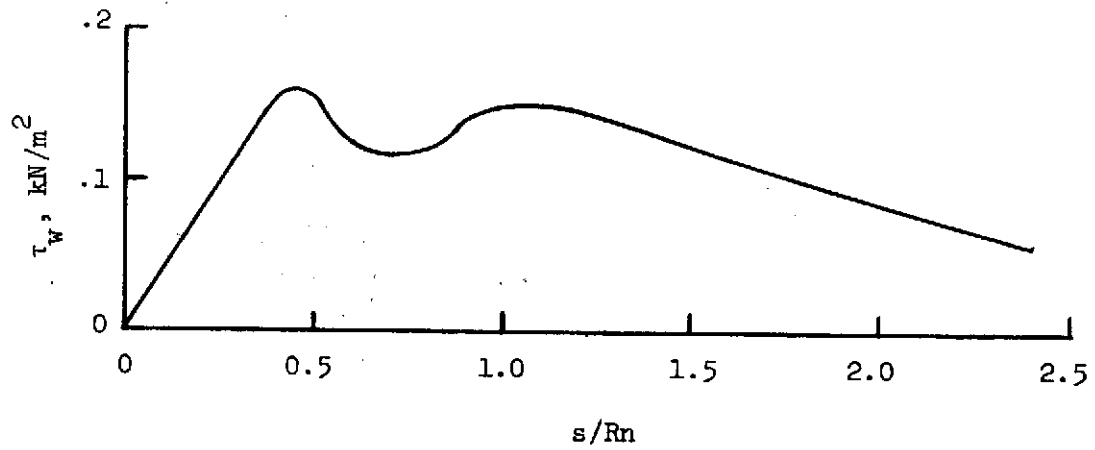
Figure 22. - Fully-coupled, radiating flow solution with steady-state ablation of carbon-phenolic heatshield for high velocity entry.



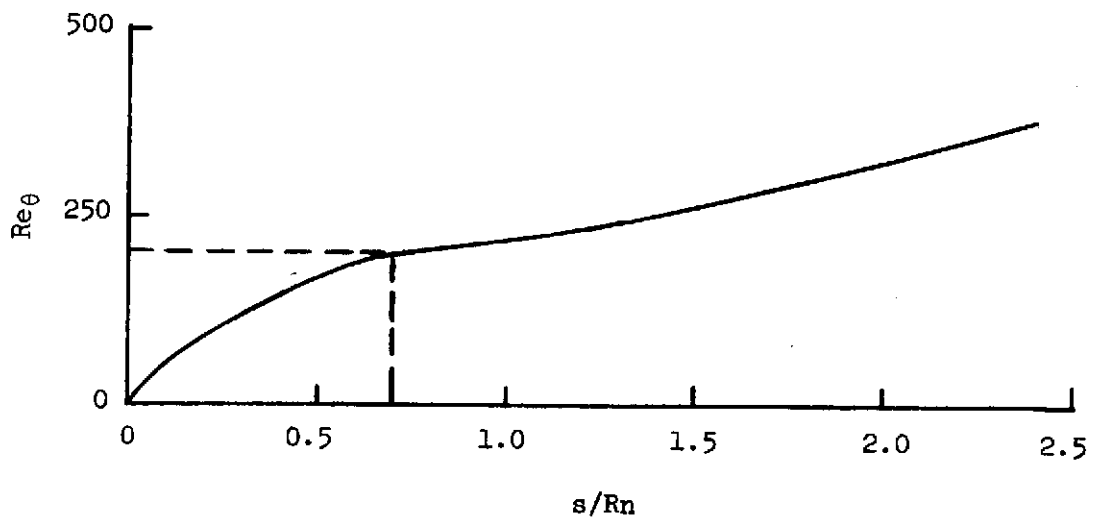
(c) Radiative and convective heating distribution.



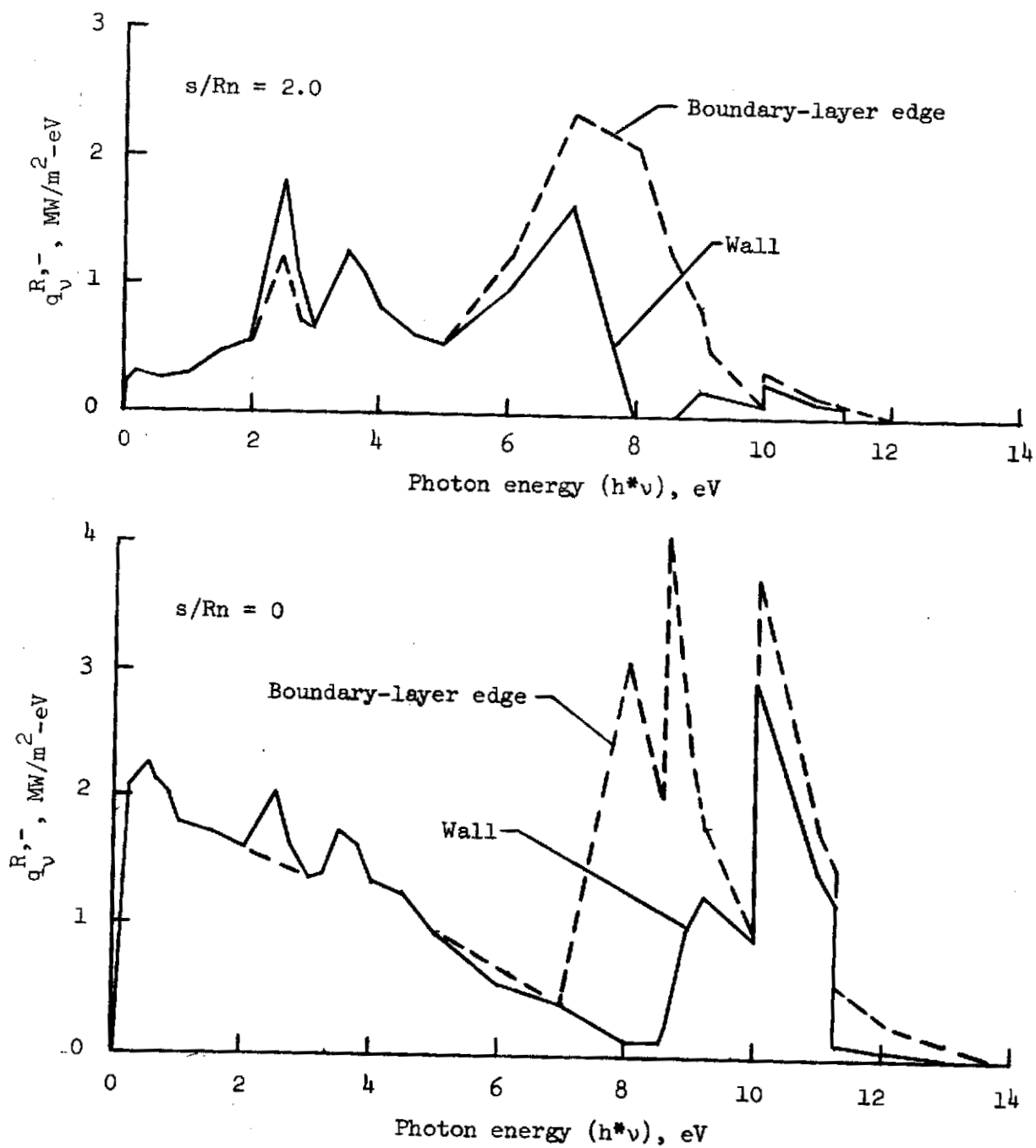
(d) Ablation rate distribution.



(e) Aerodynamic shear distribution.

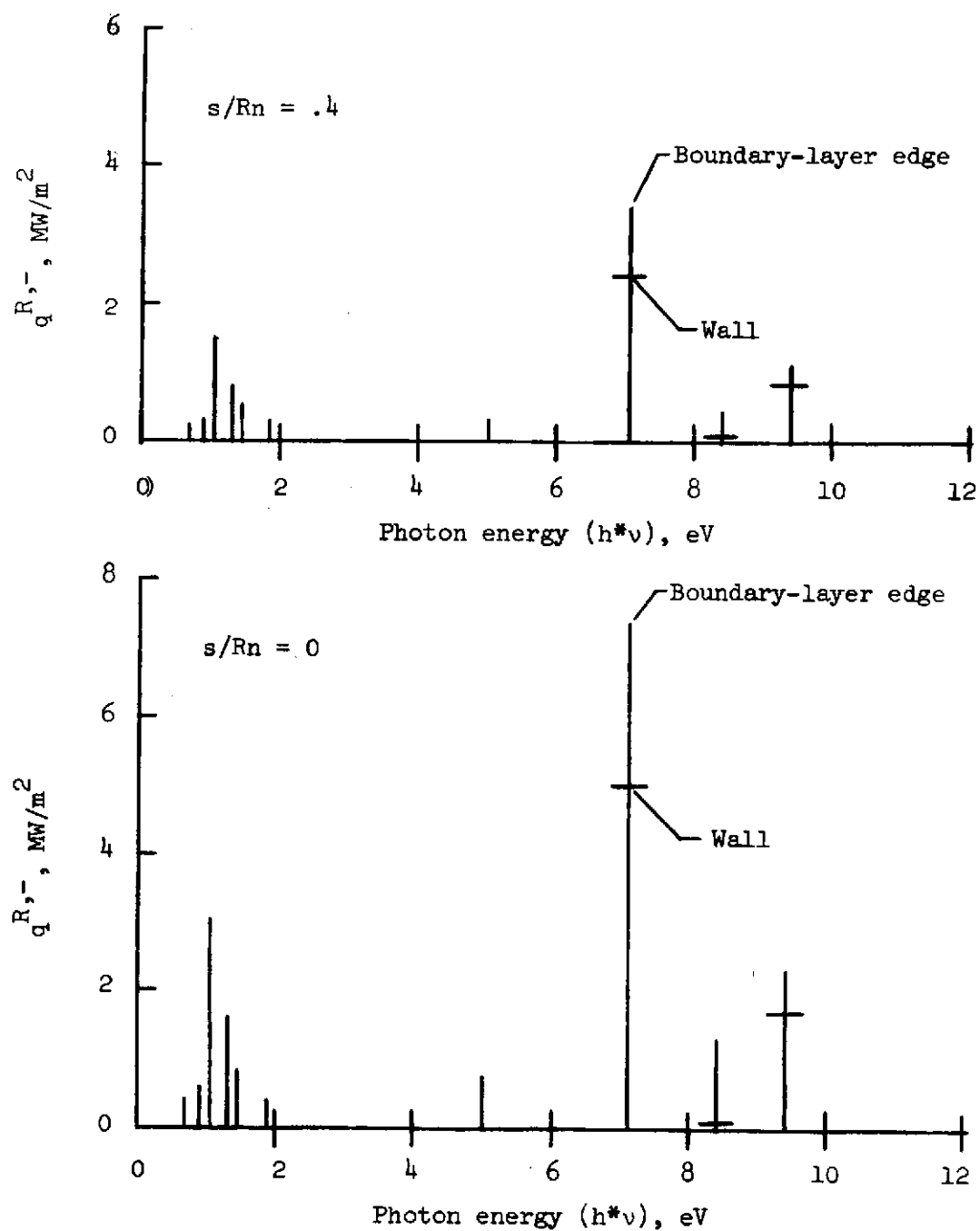


(f) Momentum thickness Reynolds number distribution.



(g) Spectral distribution of radiative heating toward the body due to continuum processes.

Figure 22. - Continued.



(h) Spectral distribution of radiative heating toward the body due to line processes.

Figure 22. - Concluded.

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APPENDIX A

UNSTEADY CHARACTERISTICS SOLUTION

A quasi, one-dimensional unsteady characteristics solution is used for the shock points in the radiating, inviscid flow field solution. The present method for this solution is similar to the methods given by Callis (ref. 7) and by Moretti and Abbett (ref. 6). A brief development of the equations is given below and further details are available in the above mentioned reports.

In the analysis, the pertinent quantities considered are the normal component of the velocity and the Y-coordinate. Since the tangential velocity is unchanged across a shock wave, the tangential velocity is not considered a pertinent quantity and the S-momentum equation is neglected. Consequentially, the equations for mass, Y-momentum, and energy are considered as quasi, one-dimensional equations modified by forcing terms on the RHS of the equations. The forcing terms are the terms in the equations which do not contain derivatives, which contain derivatives with respect to the S-coordinate, and which contain the tangential velocity.

The mass equation (eq. 17) is written as

$$\rho_T^* + A\rho_Y^* + \frac{1}{\delta} v_Y = C_1 \quad (A1)$$

where the asterisk refers to the natural logarithm of ρ that is differentiated. The Y-momentum equation (eq. 19) is written as

$$v_T + Av_Y + \frac{1}{\rho\delta} p_Y = c_2 \quad (A2)$$

In the inviscid flow field analysis, the mass equation in terms of density and the energy equation in terms of enthalpy were combined to form a new equation in terms of pressure. This combined equation replaced the mass equation; however, in the characteristics development, the combined equation is used for the energy equation. Thus, the energy equation (eq. 30) is written as

$$p_T + Ap_Y + \frac{\rho P_3}{\delta} v_Y = c_3 \quad (A3)$$

The forcing terms are

$$c_1 = - \left(\frac{u}{\lambda} \rho_S^* - \frac{B}{\rho} u_Y + \frac{1}{\lambda} u_S + \frac{C}{\rho} \right) \quad (A4)$$

$$c_2 = - \left(\frac{u}{\lambda} v_S - E \right) \quad (A5)$$

$$c_3 = - \left[\frac{u}{\lambda} p_S + P_3 \left(\frac{\rho}{\lambda} u_S + C \right) - \frac{F^R}{P_1 \rho} - P_3 B u_Y \right] \quad (A6)$$

The forcing terms are considered as known quantities and the characteristics equations and the compatibility equations are solved for by standard procedure.

The differential relations for the variables ρ , p , and v of the independent variables Y and T are

$$\rho_T^* dT + \rho_Y^* dY = d\rho^* \quad (A7)$$

$$v_T dT + v_Y dY = dv \quad (A8)$$

$$p_T dT + p_Y dY = dp \quad (A9)$$

The equations A1 to A3 and A7 to A9 written in matrix form are

$$\begin{vmatrix} \rho_T^* \\ \rho_Y^* \\ v_T \\ v_Y \\ p_T \\ p_Y \end{vmatrix} \times \begin{vmatrix} dT & dY & 0 & 0 & 0 & 0 \\ 0 & 0 & dT & dY & 0 & 0 \\ 0 & 0 & 0 & 0 & dT & dY \\ 1 & A & 0 & \frac{1}{\delta} & 0 & 0 \\ 0 & 0 & v_T & A & 0 & \frac{1}{\rho\delta} \\ 0 & 0 & 0 & \frac{\rho P_3}{\delta} & 1 & A \end{vmatrix} = \begin{vmatrix} d\rho^* \\ dv \\ dp \\ C_1 \\ C_2 \\ C_3 \end{vmatrix} \quad (A10)$$

The characteristics equations are found by setting the associated coefficient determinant equal to zero.

$$\begin{vmatrix} dT & dY & 0 & 0 & 0 & 0 \\ 0 & 0 & dT & dY & 0 & 0 \\ 0 & 0 & 0 & 0 & dT & dY \\ 1 & A & 0 & \frac{1}{\delta} & 0 & 0 \\ 0 & 0 & v_T & A & 0 & \frac{1}{\rho\delta} \\ 0 & 0 & 0 & \frac{\rho P_3}{\delta} & 1 & A \end{vmatrix} = 0 \quad (A11)$$

The solution of equation A11 is a cubic in dY/dT and the characteristics equations are

$$\frac{dY}{dT} = A \quad (A12)$$

$$\frac{dY}{dT} = A \pm \frac{\sqrt{P_3}}{\delta} \quad (A13)$$

Equation A12 is a trivial solution and is neglected. The compatibility equations are found by substituting a column of the associated coefficient determinant by the RHS of equation A10 and the determinant is set

equal to zero.

$$\begin{vmatrix} dT & dY & 0 & dp^* & 0 & 0 \\ 0 & 0 & dT & dv & 0 & 0 \\ 0 & 0 & 0 & dp & dT & dY \\ 1 & A & 0 & C_1 & 0 & 0 \\ 0 & 0 & v_T & C_2 & 0 & \frac{1}{\rho\delta} \\ 0 & 0 & 0 & C_3 & 1 & A \end{vmatrix} = 0 \quad (A14)$$

The solution of equation A14 is

$$\left(\frac{dY}{dT} - A\right) \frac{dv}{dT} + \frac{1}{\rho\delta} \frac{dp}{dT} = \left(\frac{dY}{dT} - A\right) C_2 + \frac{C_3}{\rho\delta} \quad (A15)$$

and substituting for $\frac{dY}{dT}$ from equation A13, the compatibility equations are

$$\frac{dv}{dT} \pm \frac{1}{\rho\sqrt{P_3}} \frac{dp}{dT} = C_2 \pm \frac{C_3}{\rho\sqrt{P_3}} \quad (A16)$$

From equations A13 and A16, the right running characteristic is used in the solution for the shock points and the equations are

$$\frac{dY}{dT} = A + \frac{\sqrt{P_3}}{\delta} \quad (A17)$$

$$\frac{dv}{dT} + \frac{1}{\rho\sqrt{P_3}} \frac{dp}{dT} = C_2 + \frac{C_3}{\rho\sqrt{P_3}} \quad (A18)$$

APPENDIX B

STAGNATION-POINT SOLUTION FOR RADIATING,
INVISCID FLOW FIELD

The equations for the radiating, inviscid flow field around an axisymmetric blunt body are reduced so that they are applicable for only a stagnation-point solution. Note, the equations presented in this section are not used for the stagnation point of the complete body solution but are used in a separate program when only a stagnation-point solution is required.

The following conditions, from symmetry of the body, exist at the stagnation line.

$$\left. \begin{array}{lll} u = 0 & v_S = 0 & \beta = 0 \\ u_Y = 0 & h_S = 0 & \theta = \pi/2 \\ \rho_S = 0 & p_S = 0 & K = 1 \end{array} \right\} \quad (B1)$$

With these conditions, the equations for mass, Y-momentum, and energy (eqs. 30, 19, and 20) become

mass:

$$p_T = - \left[A p_Y + P_3 \left(\frac{2\rho}{\lambda} u_S + \frac{\rho}{\lambda} v_Y + \frac{2\rho v}{\lambda} \right) - \frac{F^R}{P_1 \rho} \right] \quad (B2)$$

Y-momentum:

$$v_T = - \left(A v_Y + \frac{1}{\rho \delta} p_Y \right) \quad (B3)$$

energy:

$$h_T = - \left(A h_Y - \frac{1}{\rho} p_T - \frac{A}{\rho} p_Y + \frac{F^R}{\rho} \right) \quad (B4)$$

The equation for S-momentum (eq. 18) is degenerate at the stagnation line; but, by differentiation of equation 18 with respect to S and applying the symmetry conditions the following equation is obtained

S-momentum:

$$u_{ST} = -\left(Au_{SY} + \frac{1}{\lambda} u_S^2 + \frac{v}{\lambda} u_S + \frac{1}{\rho\lambda} p_{SS}\right) \quad (B5)$$

The S derivative of the tangential velocity, u_S , is used in the stagnation-point solution rather than the tangential velocity. The parameters in the equations are

$$A = \frac{v}{\delta} - \frac{Y}{\delta} \delta_T \quad (B6)$$

$$F^R = \frac{1}{\delta} q_Y^R + \frac{2}{\lambda} q^R \quad (B7)$$

$$\lambda = 1 + \delta Y \quad (B8)$$

In order to solve the set of equation an expression is needed for p_{SS} .

An approximation for this parameter is taken from Newtonian theory

$$p_{SS} = -2 \quad (B9)$$

The characteristic equation and the compatibility equation for the unsteady characteristic solution at the shock location become

$$\frac{dY}{dT} = A + \frac{\sqrt{P_3}}{\delta} \quad (B10)$$

$$\frac{dv}{dT} + \frac{1}{\rho\sqrt{P_3}} \frac{dp}{dT} = \frac{C_3}{\rho\sqrt{P_3}} \quad (B11)$$

where

$$C_3 = -\left[\frac{2\rho}{\lambda} P_3 (u_S + v) - \frac{F^R}{P_1 \rho}\right] \quad (B12)$$

The equations presented in this section are non-dimensionalized by the expression given in equation 14. Indeterminate terms were evaluated by the L'Hospital rule.

APPENDIX C

COMPUTER PROGRAM FOR RADIATING, INVISCID
FLOW FIELD SOLUTION

A program listing and a description of inputs are given for the computer program developed for the radiating, inviscid flow field solution. The program is written in FORTRAN IV language for CDC 6000 series computers. In order to run the program, all variables have to be initially set to zero by means external to the program. A system subroutine called by a SETCORE control card is used for this external initialization for the computer system used in the present study.

The inputs and printed outputs for a sample case are also given. The sample case is a radiating solution for a .97 CO₂ - .03 N₂ (by volume) gas composition. The body shape is a hemispherical nose, .3048 meter radius, with a 60° half-angle, conical afterbody.

PROGRAM LISTING

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```

252  S2 = UC2,1175C21
253  RT [R1:1] = -1*VAL1,11*VAL1,11*RHOD1,13*E22*WV3/DELTA(13)11
254  I = CDD1,11*E1/11,11*WDD1,11,11
255  47FL1 = -E1/11,11*11*RHOD1,13
256  RHOD1(1,1) = CDD1,13*E1/11,11*11*RHOD1,133*OPT1,13/13/E1(1,1)11
257  LVA(1,11*VAL1,11,11)
258  GO 91
259  DO 91 RT=2,KV51
260  S2 = U2P,KV52/2
261  IPRV=EQ,KV51 GO TO 9101
262  Y3 = IVEL,KV11-VEL,KV1/DELTA(KV1
263  V4 = IPI,KV11-DEL,KV1/DELTA(KV1
264  Y5 = IDEL,KV1/DELTA(KV1
265  IDELVB(KV1
266  GO 9102
267  9101 CONTINUE
268  V3 = EYE,KV11*DELTA(KV1/VEL,1,KV11-DELTA(3(KV1/VEL,KV1-133)11
269  IDELVB(KV1
270  V4 = IPI,KV11+DELTA(KV1/DELTA(KV1,KV11-DELTA(3(KV1/DELTA(KV1,KV1-133)11
271  IDELVB(KV1
272  V5 = IDEL,KV1/DELTA(KV1,KV11+DELTA(KV1/DELTA(KV1,KV1-13)11-DEL(KV1-23)11
273  IDELVB(KV1
274  9102 CONTINUE
275  G4 = IVEL,KV11+VEL,KV1/DELTA(13)11/DELTA(13
276  S1 = 1.0 - (K11*VEL/DELTA(13)11
277  OPT1(LAY) = G4
278  V11(KV1) = IGA4/KV11 - IGA4/RHOD1,KV1/DELTA(13)11
279  IPI(KV1) = ICG4/KV11 - IGA4/KV11+VEL,KV1/DELTA(13)11/RHOD1,KV1/DELTA(13)11
280  IPI(RHOD1,KV1/DELTA(13)11)11/2 - RHOD1(RHOD1,KV1/DELTA(13)11)11
281  21DEL(OPT1/PI/PI,KV1/PI,RHOD1,KV1)11
282  HPI(KV1) = ICG4/KV11 - IPI(KV1/DELTA(KV1,KV1-13)11/RHOD1,KV1/DELTA(13)11
283  RHOD1(KV1) = ICG4/KV11 - IPI(KV1/DELTA(KV1,KV1-13)11/RHOD1,KV1/DELTA(13)11
284  IPI(KV1) = IPI(KV1/DELTA(KV1,KV1-13)11/RHOD1,KV1/DELTA(13)11
285  91 CONTINUE
286  DO 90 K=2,K5F
287  K3 = IDELVB(13)11*VEL,KV11 - IDELVB(13)11*VEL,KV1,23)11 -VKV5,13*DELVB(13)11
288  IPRV5,KV5 GO TO 191
289  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
290  IDEL5(KV5)
291  S = IPI(KV5,11)11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
292  IDEL5(KV5)
293  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
294  IDEL5(KV5)
295  GO TO 291
296  191 CONTINUE
297  S2 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-2,13)11 -UKV5-2,13)11
298  IDEL5(KV5)
299  S = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11 -PI(KV5-2,13)11
300  IDEL5(KV5)
301  S = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11 -PI(KV5-2,13)11
302  IDEL5(KV5)
303  291 CONTINUE
304  S4 = I(RHOD1,KV1/DELTA(13)11)11*DELTA(13)11/DELTA(13)11/DELTA(13)11
305  IPRV5,KV5 GO TO 191
306  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
307  IDEL5(KV5)
308  S = IPI(KV5,11)11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
309  IDEL5(KV5)
310  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
311  IDEL5(KV5)
312  90 CONTINUE
313  DO 101 K=2,K5F
314  DO 101 K=2,K5F
315  IPRV5,KV5 GO TO 1301
316  S2 = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
317  IDEL5(KV5)
318  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
319  IDEL5(KV5)
320  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
321  IDEL5(KV5)
322  S = IUKV5,KV11+DELTA(5)11*SPRINKS,13)11 -IDEL5(KV5)11*SPRINKS-1,133)11
323  IDEL5(KV5)
324  GO TO 2101
325  1301 CONTINUE
326  S2 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11 -UKV5-2,KV5)11
327  IDEL5(KV5)
328  S3 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11 -VKV5-2,KV5)11
329  IDEL5(KV5)
330  S4 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11 -PI(KV5-2,KV5)11
331  IDEL5(KV5)
332  S5 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11 -HEMS-2,KV5)11
333  IDEL5(KV5)
334  2101 CONTINUE
335  IPRV=EQ,KV51 GO TO 191
336  Y3 = IVEL,KV11-VEL,KV1/DELTA(KV1
337  V4 = IPI,KV11-DEL,KV1/DELTA(KV1
338  Y5 = IDEL,KV1/DELTA(KV1
339  IDELVB(KV1
340  V3 = IDEL,KV1/DELTA(KV1,KV11+DELTA(KV1/DELTA(KV1,KV1-13)11
341  IDELVB(KV1
342  GO TO 1912
343  1912 CONTINUE
344  92 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-2,13)11 -UKV5,KV1-23)11
345  IDELVB(KV1
346  Y3 = IVEL,KV11+DELTA(KV1/DELTA(KV1,KV11-DELTA(3(KV1/DELTA(KV1,KV1-133)11
347  IDELVB(KV1
348  V4 = IPI,KV11+DELTA(KV1/DELTA(KV1,KV11-DELTA(3(KV1/DELTA(KV1,KV1-133)11
349  IDELVB(KV1
350  Y5 = IDEL,KV1/DELTA(KV1,KV11+DELTA(KV1/DELTA(KV1,KV1-13)11-HEMS,KV1-23)11
351  IDELVB(KV1
352  1912 CONTINUE
353  G1 = I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11
354  I-DEL5(KV5)11*SPRINKS,13)11+DELTA(5)11*SPRINKS-1,13)11
355  R = RHOD1 = IVEL,KV1/DELTA(13)11*DELTA(13)11/DELTA(13)11
356  C1 = IPRV/DELTA(13)11*DELTA(13)11/DELTA(13)11
357  S4 = IVEL,KV11-VEL,KV1/DELTA(KV1,KV11+DELTA(KV1/DELTA(KV1,KV1-13)11
358  C5 = IPRV/DELTA(13)11*DELTA(13)11/DELTA(13)11
359  IPRV5,KV5 GO TO 1912
360  IPRV5,KV5 GO TO 1912
361  IPRV5,KV5 GO TO 1912
362  IPRV5,KV5 GO TO 1912
363  IPRV5,KV5 GO TO 1912
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367  IPRV5,KV5 GO TO 1912
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380  IPRV5,KV5 GO TO 1912
381  IPRV5,KV5 GO TO 1912
382  IPRV5,KV5 GO TO 1912
383  IPRV5,KV5 GO TO 1912
384  IPRV5,KV5 GO TO 1912
385  IPRV5,KV5 GO TO 1912

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1601 2201 APEELJ1=0.
1602 APEELJ1=1.0
1603 2204 CONTINUE
1604 IG=100
1605 CALL REKEYIN(APEEL,11,3,0,8,11,1,0,0,14)
1606 CT=CT+1
1607 IF(CT-100) 222,2209,2210
1608 2204 100=2
1609 2210 IF(IG) 2217,2221,2221
1610 2212 IF(1NV) 2218,2213,222
1611 2213 1NV = -1
1612 PTHWPL1=-5
1613 50 2218 1=1,N
1614 IF(EPC(1)) 2219,2216,2215
1615 2216 V(11)=V(11)+P(11)
1616 V(11)=ABS(V(11))
1617 2215 CONTINUE
1618 GO TO 111
1619 2216 CONTINUE
1620 2217 1TS=1000
1621 GO TO 111
1622 2217 1TS=1000
1623 2217 1TS=1000
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1625 2217 1TS=1000
1626 2217 1TS=1000
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1827 2217 1TS=1000
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1871      IF (IT1) 405,460,460
1872      460 IF (IPC1)=1 405,465,465
1873      465 CONTINUE
1874      475
1875      IF (C1)=1
1876      480
1877      485
1878      490
1879      495
1880      500
1881      505
1882      510
1883      515
1884      520
1885      525
1886      530
1887      535
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1889      545
1890      550
1891      555
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1906      630
1907      635
1908      640
1909      645
1910      650
1911      655
1912      660
1913      665
1914      670
1915      675
1916      680
1917      685
1918      690
1919      695
1920      700
1921      705
1922      710
1923      715
1924      720
1925      725
1926      730
1927      735
1928      740
1929      745
1930      750
1931      755
1932      760
1933      765
1934      770
1935      775
1936      780
1937      785
1938      790
1939      795
1940      800
1941      805
1942      810
1943      815
1944      820
1945      825
1946      830
1947      835
1948      840
1949      845
1950      850
1951      855
1952      860
1953      865
1954      870
1955      875
1956      880
1957      885
1958      890
1959      895
1960      900
1961      905
1962      910
1963      915
1964      920
1965      925
1966      930
1967      935
1968      940
1969      945
1970      950
1971      955
1972      960
1973      965
1974      970
1975      975
1976      980
1977      985
1978      990
1979      995
1980      1000
1981      1005
1982      1010
1983      1015
1984      1020
1985      1025
1986      1030
1987      1035
1988      1040
1989      1045
1990      1050
1991      1055
1992      1060
1993      1065
1994      1070
1995      1075
1996      1080
1997      1085
1998      1090
1999      1095
2000      1100
2001      1105
2002      1110
2003      1115
2004      1120
2005      1125

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DESCRIPTION OF INPUT

CARD SET 1 - CONTROL CARD AND CASE IDENTIFICATION

CARD 1, FORMAT(11,15A5)

FIELD 1, (COLUMNS 1-7) KQS

COLUMN 1 DETERMINES IF CHANGES ARE TO BE MADE FOR PARAMETERS IN NAMELIST NAME2. SEE CARD SET 4.
0 NO CHANGES IN NAME2
1 CHANGES IN NAME2
COLUMN 2 DETERMINES IF MASS BALANCE OPTION TO BE USED. SEE ALSO THE PARAMETERS LSC AND CGS IN NAME2 OF CARD SET 4.
0 MASS BALANCE NOT USED
1 MASS BALANCE USED
COLUMN 3 DETERMINES IF RADIATIVE TRANSPORT IS CONSIDERED.
0 NO RADIATIVE TRANSPORT
1 RADIATIVE TRANSPORT
COLUMN 4 DETERMINES IF MOLECULES ARE TO BE INCLUDED IN RADIATIVE TRANSPORT.
0 MOLECULES INCLUDED (SEE CARD SET 7)
1 MOLECULES NOT INCLUDED
COLUMN 5 DETERMINES HOW THE LINE CONTRIBUTIONS ARE CALCULATED IN THE RADIATIVE TRANSPORT.
0 INCLUDE LINES, WEAK LINES TREATED APPROXIMATELY
1 INCLUDE LINES IN FULL DETAIL
2 EXCLUDE LINES

FIELD 2, (COLUMNS 8-80) CASE IDENTIFICATION (ALPHANUMERIC).

CARD SET 2 - FREESTREAM CONDITIONS, NUMBER OF NODES, NOSE RADIUS.

CARDS AS REQUIRED, NAMELIST FROMAT (NAME1)

VF FREESTREAM VELOCITY, METERS PER SECOND
RHO F FREESTREAM DENSITY, KILOGRAMS PER CUBIC METER
PF FREESTREAM PRESSURE, NEWTONS PER SQUARE METER
HF FREESTREAM ENTHALPY, JOULES PER KILOGRAM
RN BODY RADIUS AT STAGNATION POINT (NOSE RADIUS), METERS
KYS NUMBER OF NODES THROUGH LAYER INCLUDING WALL AND SHOCK LOCATIONS. MAXIMUM OF 11.
Y(1) ARRAY OF NONDIMENSIONAL LOCATION OF NODES THROUGH LAYER (1=1,KYS). Y(1)=0 AT WALL AND Y(KYS)=1 AT SHOCK.
KSE NUMBER OF NODES AROUND BODY. MAXIMUM OF 15.

CARD SET 3 - SPECIFICATION OF BODY SHAPE AT NODES AROUND BODY.

CARDS AS REQUIRED AND EQUAL TO KSE, FORMAT(15F15.7). ONE CARD FOR EACH BODY LOCATION. BEGIN AT S=0. LENGTHS TO BE NONDIMENSIONAL BY RN AND ANGLES IN RADIANS.
COLUMN 1-15 THETA, BODY ANGLE
COLUMN 16-30 S, DISTANCE ALONG BODY FROM STAGNATION POINT
COLUMN 31-45 R0, RADIUS OF BODY PERPENDICULAR TO AXIS
COLUMN 46-60 X0, AXIAL DISTANCE FROM STAGNATION POINT
COLUMN 61-75 K, BODY CURVATURE TERM

CARD SET 4 - CONVERGENCE TERMS, OUTPUT, MASS BALANCE TERMS, ETC.

SKIP THIS SET IF KQS(1)=0.

CARDS AS REQUIRED, NAMELIST FROMAT (NAME2)

VALUES FOR THE PARAMETERS IN NAME2 ARE SET IN THE PROGRAM, BUT ARE NOT NECESSARILY GOOD VALUES FOR ALL PROBLEMS. TO CHANGE ANY OF THEM, SET KQS(1)=1 AND READ IN THE VALUES FOR THE PARAMETERS WHICH ARE TO BE CHANGED. FOR NO CHANGES, SET KQS(1)=0 AND SKIP THIS CARD SET.
DF1 MULTIPLY OF CORRELATION FOR INITIAL SHOCK STAND-OFF DISTANCE.
TIFRAC MULTIPLY OF VALUE FOR GETTING TIME STEP. MUST BE LESS THAN 1.0.
IKSC THE ITERATION TO BEGIN CHECKING FOR CONVERGENCE.
CCI CONVERGENCE CRITERION. THE CHANGE IN ENTHALPY BETWEEN ITERATIONS, $(H_{IN+1} - H_{IN}) / H_{IN}$, MUST BE LESS THAN CCI FOR CONVERGENCE.
CCF FRACTION OF NODES POINTS ALONG WALL, SHOCK, AND LAYER MIDPOINT FOR WHICH THE CONVERGENCE CRITERION MUST BE MET. VALUE FOR CCF MUST BE LESS THAN 1.0.
ICC CONSECUTIVE NUMBER OF ITERATIONS FOR WHICH THE ERROR CRITERION MUST BE MET.
IKSF MAXIMUM NUMBER OF ITERATIONS TO BE ALLOWED. CARDS ARE PUNCHED TO PICK UP CASE BY ANOTHER RUN. SEE CARD SET 5.
IOUTN PRINTED OUTPUT IS GIVEN FOR CONVERGED SOLUTION. FOR INTERMEDIATE OUTPUT, IOUTN IS THE NUMBER OF ITERATIONS BETWEEN OUTPUT.

IOUTF IF THE INITIAL VALUES (FIRST GUESS VALUES) ARE TO BE PRINTED, SET THE VALUE OF IOUTF TO IOUTN MINUS ONE. IF NOT, SET IOUTF EQUAL TO ZERO.
ITC NUMBER OF ITERATIONS BETWEEN CALCULATION OF THERMODYNAMIC PROPERTIES.
IRC NUMBER OF ITERATIONS BETWEEN CALCULATION OF RADIATIVE TRANSPORT.
IKSU(1) ARRAY FOR VALUES OF ITERATIONS AT WHICH THE VALUES FOR ITC AND IRC CAN BE CHANGED DURING SOLUTION. MAXIMUM OF 5.
ITCU(1) ARRAY OF VALUES FOR ITC WHICH CORRESPONDS TO IKSU(1).
IRCUI(1) ARRAY OF VALUES FOR IRC WHICH CORRESPONDS TO IKSU(1).
NCTS IF NCTS EQUALS ZERO, THE TIME STEP CAN VARY AROUND BODY AND SOLUTION WILL GENERALLY CONVERGE FASTER. IF NCTS IS NOT EQUAL TO ZERO, THE MINIMUM TIME STEP IS USED AT ALL BODY LOCATIONS. CRITERION TO BE MET BEFORE MASS BALANCE OPTION CAN BE USED. THE SHOCK VELOCITY AT THE STAGNATION POINT (DELTA T(1)) MUST BE LESS THAN THE SET VALUE FOR CGS BEFORE THE MASS BALANCE OPTION CAN BE USED FOR CALCULATING THE SHOCK STAND-OFF DISTANCE AROUND BODY. THIS IS BECAUSE THE SHOCK STAND-OFF DISTANCE AT THE STAGNATION POINT MUST BE CONVERGED BEFORE USING THE MASS BALANCE OPTION.
LSC THE MINIMUM ITERATION AT WHICH THE MASS BALANCE OPTION CAN BE USED.

CARD SET 5 - INPUT FOR THE PICK UP OF A SOLUTION.

THE PROGRAM IS WRITTEN SO THAT A SOLUTION CAN BE PICKED UP FROM A SPECIFIED ITERATION.

FOR A NEW CASE, A BLANK CARD IS INSERTED FOR THIS CARD SET.

IF THE NUMBER OF ITERATIONS REACHES THE VALUE SET FOR IKSF (CARD SET 4) THEN A CONVERGED SOLUTION WAS NOT OBTAINED WITHIN THE SPECIFIED NUMBER OF ITERATIONS AND A SET OF CARDS ARE PUNCHED OUT. THIS SET OF PUNCHED OUTPUT CAN BE INSERTED FOR CARD SET 5 AND THE SOLUTION CAN BE PICKED UP BY ANOTHER RUN AT THAT ITERATION. FOR A PICK UP CASE, IKSF AND IKSC (CARD SET 4) MUST BE GREATER THAN THE VALUE OF IKSF FROM THE PRIOR RUN.

CARD SET 6, 7 AND 8.

THESE CARD SETS ARE THE CHEMISTRY INPUTS FOR THE CHEMICAL EQUILIBRIUM CALCULATIONS AND THE BASIC RADIATION AND SPECTROSCOPIC DATA FOR THE RADIATIVE TRANSPORT CALCULATIONS. THE INPUTS FOR THE CARD SETS ARE DESCRIBED IN THE REPORT - NICOLET, W.E. USER'S MANUAL FOR THE GENERALIZED RADIATION TRANSFER CODE (RAD/EQUIL). AEROTHERM REPORT NUMBER UM-69-9. AEROTHERM CORPORATION, MOUNTAIN VIEW, CALIFORNIA.
CARD SET 6 IS READ IN THE SUBROUTINE INPUT. CARD SETS 7 AND 8 ARE READ IN SUBROUTINE RADIN.

CARD SET 6 - CHEMISTRY INPUT.

THE INPUTS FOR THIS CARD SET ARE DESCRIBED IN GROUP 3 AND GROUP 4 OF DECK B IN THE REPORT BY NICOLET.

MODIFICATIONS

GROUP 3 IN COLUMNS 25-30 ON EACH ELEMENT CARD, EXCEPT ELECTRON, GIVE THE ELEMENTAL MASS FRACTION FOR THAT ELEMENT. (AS A NEGATIVE NUMBER) NOTE: MASS FRACTION OF ELEMENT.
GROUP 4 THE THERMODYNAMIC INPUT DATA (3 CARD SET) FOR THE ATOMIC SPECIES OF THE ELEMENTS MUST BE ARRANGED FIRST IN THE SAME ORDER AS GIVEN BY GROUP 3. THE DATA FOR THE ELECTRON AND IONIZED SPECIES ARE STILL AT THE END. CONDENSED SPECIES ARE NOT ALLOWED. MAXIMUM NUMBER OF SPECIES IS 40. A BLANK CARD MUST BE PLACED AT THE END OF GROUP 4 AS STATED BY NICOLET.

CARD SET 7 - CONTINUUM CONTRIBUTIONS TO RADIATIVE TRANSPORT. SKIP THIS CARD SET IF KQS(3)=0.

THE INPUTS FOR THIS CARD SET ARE DESCRIBED IN GROUP 6, CARD 3 OF DECK B IN THE REPORT BY NICOLET.
IDENTIFIES WHICH CONTINUUM CONTRIBUTIONS ARE TO BE INCLUDED IN THE CALCULATION OF RADIATIVE TRANSPORT.

CARD SET 8 - RADIATION AND SPECTROSCOPIC DATA. SKIP THIS CARD SET IF KQS(3)=0.

THE INPUTS FOR THIS CARD SET ARE DESCRIBED IN DECK A (ENTIRE DECK) IN THE REPORT BY NICOLET.

| | | | | | | | | | |
|----|---------|---------|------------|-----|-------|--------|---------|----------|-------|
| 5 | 0.7525 | 0.0149 | 446E-21 | 151 | 11 | 10.187 | 0.1510 | 653E-22 | 226 |
| 22 | 0.844 | 0.0808 | 412E-21 3. | 152 | 25 | 10.196 | 0.4162 | | 227 |
| 21 | 0.852 | 0.0687 | 108E-21 | 153 | 1 | 10.137 | 0.1840 | 621E-22 | 228 |
| 5 | 0.875 | 0.0366 | 780E-21 4. | 154 | 17 | 10.401 | 0.30719 | 220E-20 | 229 |
| 15 | 0.8840 | 0.1570 | 367E-21 | 155 | 17 | 10.405 | 0.0495 | 109E-19 | 230 |
| 4 | 0.9158 | 0.00447 | 730E-22 | 156 | 3 | 10.418 | 0.0225 | 537E-20 | 231 |
| 6 | 0.9304 | 0.0253 | 387E-20 2. | 157 | 2 | 10.493 | 0.0187 | 446E-20 | 232 |
| 6 | 0.965 | 0.0262 | 387E-20 4. | 158 | 3 | 10.555 | 0.0131 | 796E-20 | 233 |
| 16 | 0.991 | 0.0805 | 309E-20 2. | 159 | 2 | 10.619 | 0.0533 | 312E-21 | 234 |
| 23 | 1.019 | 0.0329 | 205E-19 4. | 160 | 3 | 10.682 | 0.00810 | 268E-19 | 235 |
| 5 | 1.0355 | 0.0735 | 331E-21 7. | 161 | 17 | 10.714 | 0.0298 | 690E-19 | 236 |
| 22 | 1.079 | 0.1008 | 320E-21 8. | 162 | 3 | 10.757 | 0.00518 | 437E-19 | 237 |
| 15 | 1.0980 | 0.7490 | 344E-21 | 163 | 10 | 10.761 | 0.1200 | 653E-22 | 238 |
| 14 | 1.1320 | 0.2010 | 367E-21 | 164 | 18 | 10.873 | 0.705 | 630E-21 | 239 |
| 21 | 1.163 | 0.474 | 108E-21 | 165 | 17 | 10.875 | 0.0155 | 130E-18 | 240 |
| 22 | 1.224 | 0.0285 | 262E-21 | 166 | 17 | 10.887 | 0.00194 | 119E-19 | 241 |
| 5 | 1.2610 | 0.118 | 312E-21 3. | 167 | 1 | 10.927 | 0.4940 | 148E-23 | 242 |
| 4 | 1.3190 | 0.1833 | 984E-22 | 168 | 17 | 10.986 | 0.011 | 253E-19 | 243 |
| 21 | 1.326 | 0.206 | 138E-21 3. | 169 | 11 | 11.007 | 0.0185 | 367E-21 | 244 |
| 14 | 1.3380 | 0.9130 | 342E-21 | 170 | 17 | 11.061 | 0.00659 | 477E-18 | 245 |
| 5 | 1.3677 | 0.0387 | 292E-21 | 171 | 3 | 11.200 | 0.0200 | 446E-21 | 246 |
| 4 | 1.4380 | 0.256 | 874E-22 3. | 172 | 2 | 11.293 | 0.0418 | 293E-20 | 247 |
| 13 | 1.4670 | 0.9500 | 965E-22 | 173 | 3 | 11.310 | 0.0254 | 323E-21 | 248 |
| 21 | 1.487 | 0.0405 | 218E-21 | 174 | 7 | 11.424 | 0.2760 | 143E-23 | 249 |
| 5 | 1.5527 | 0.0030 | 293E-20 | 175 | 2 | 11.509 | 0.0259 | 832E-20 | 250 |
| 12 | 1.594 | 1.0300 | 709E-22 | 176 | 2 | 11.776 | 0.0220 | 796E-20 | 251 |
| 4 | 1.6630 | 0.0923 | 948E-22 | 177 | 11 | 11.806 | 0.0049 | 145E-20 | 252 |
| 15 | 1.767 | 0.0226 | 275E-20 3. | 178 | 09 | 11.852 | 0.0199 | 367E-21 | 253 |
| 22 | 1.814 | 0.0039 | 350E-20 5. | 179 | 2 | 11.874 | 0.0091 | 268E-19 | 254 |
| 5 | 1.8357 | 0.00566 | 293E-20 5. | 180 | 2 | 11.948 | 0.00575 | 437E-19 | 255 |
| 26 | 1.888 | 0.6407 | | 181 | 3 | 12.000 | 0.0269 | 290E-20 | 256 |
| 14 | 2.015 | 0.0256 | 275E-20 3. | 182 | 09 | 12.067 | 0.0218 | 346E-21 | 257 |
| 26 | 2.549 | 0.1193 | | 183 | 25 | 12.084 | 0.0791 | | 258 |
| 4 | 2.925 | 0.0070 | 106E-20 3. | 184 | 11 | 12.160 | 0.0019 | 128E-20 | 259 |
| 13 | 3.0 | 0.010 | 810E-21 2. | 185 | 10 | 12.181 | 1.05 | 159E-22 | 260 |
| 12 | 3.167 | 0.00826 | 520E-21 | 186 | 3 | 12.316 | 0.0156 | 696E-20 | 261 |
| 4 | 3.4724 | 0.00861 | 452E-20 3. | 187 | 10 | 12.404 | 0.0461 | 653E-22 | 262 |
| 12 | 3.7110 | 0.0143 | 110E-20 | 188 | 2 | 12.414 | 0.0574 | 390E-21 | 263 |
| 19 | 5.002 | 0.0678 | 113E-21 | 189 | 2 | 12.511 | 0.0279 | 337E-21 | 264 |
| 18 | 6.424 | 0.07290 | 113E-21 | 190 | 09 | 12.521 | 0.0775 | 633E-22 | 265 |
| 19 | 7.013 | 0.01410 | 500E-21 | 191 | 09 | 12.651 | 0.00524 | 145E-20 | 266 |
| 19 | 7.078 | 0.0748 | 262E-21 | 192 | 25 | 12.745 | 0.02899 | 4.59E-20 | 267 |
| 3 | 07.111 | 0.0634 | 912E-22 | 193 | 1 | 12.877 | 0.0233 | 446E-21 | 268 |
| 15 | 7.461 | 0.105 | 673E-22 | 194 | 1 | 13.004 | 0.1320 | 294E-21 | 269 |
| 19 | 7.717 | 0.00524 | 220E-20 | 195 | 17 | 13.119 | 0.379 | 101E-21 | 270 |
| 19 | 7.721 | 0.0367 | 109E-19 | 196 | 2 | 13.190 | 0.0489 | 290E-20 | 271 |
| 17 | 7.947 | 0.283 | 208E-22 | 197 | 2 | 13.508 | 0.0291 | 696E-19 | 272 |
| 19 | 8.030 | 0.70457 | 690E-19 | 198 | 7 | 13.543 | 0.1610 | 950E-24 | 273 |
| 19 | 8.191 | 0.0116 | 130E-18 | 199 | 18 | 13.601 | 0.295 | 159E-22 | 274 |
| 19 | 8.203 | 0.00147 | 119E-19 | 200 | 1 | 13.677 | 0.0957 | 293E-20 | 275 |
| 19 | 8.302 | 0.00831 | 538E-18 | 201 | 1 | 13.993 | 0.0584 | 532E-20 | 276 |
| 2 | 08.3021 | 0.0740 | 912E-22 | 202 | 1 | 14.160 | 0.0362 | 796E-20 | 277 |
| 18 | 8.368 | 0.011 | 214E-21 | 203 | 1 | 14.257 | 0.0212 | 268E-19 | 278 |
| 19 | 8.377 | 0.00501 | 677E-18 | 204 | 1 | 14.332 | 0.0138 | 437E-19 | 279 |
| 18 | 8.433 | 0.0142 | 500E-21 | 205 | | | | | 280 |
| 18 | 8.474 | 0.0625 | 248E-21 | 206 | 48 | 0.02 | 0.1 | 0.2 | 0.5 |
| 3 | 08.781 | 0.0435 | 661E-22 | 207 | 1.00 | 1.50 | 2.00 | 0.6 | 0.8 |
| 18 | 9.137 | 0.00526 | 220E-20 | 208 | 3.25 | 3.50 | 3.75 | 2.5 | 2.75 |
| 18 | 9.141 | 0.0362 | 109E-19 | 209 | 6.00 | 7.00 | 8.00 | 4.0 | 4.5 |
| 3 | 09.301 | 0.0164 | 446E-21 | 210 | 9.00 | 9.19 | 9.21 | 8.50 | 8.52 |
| 17 | 9.332 | 0.203 | 557E-23 | 211 | 10.81 | 11.00 | 11.25 | 10.01 | 10.79 |
| 3 | 09.394 | 0.0119 | 259E-21 | 212 | 12.18 | 12.20 | 12.99 | 11.27 | 11.99 |
| 19 | 9.450 | 0.0218 | 490E-19 | 213 | 15.59 | 15.61 | 14.29 | 13.00 | 13.59 |
| 3 | 09.460 | 0.0360 | 336E-21 | 214 | | | | 14.31 | 14.55 |
| 09 | 9.5010 | 0.0471 | 548E-22 | 215 | | | | 15.00 | |
| 18 | 9.611 | 0.0114 | 130E-18 | 216 | | | | | |
| 18 | 9.623 | 0.00143 | 119E-19 | 217 | | | | | |
| 17 | 9.697 | 0.01950 | 500E-21 | 218 | | | | | |
| 17 | 9.698 | 0.0038 | 239E-21 | 219 | | | | | |
| 17 | 9.709 | 0.0767 | 238E-21 | 220 | | | | | |
| 18 | 9.722 | 0.0001 | 538E-18 | 221 | | | | | |
| 18 | 9.797 | 0.00488 | 677E-18 | 222 | | | | | |
| 17 | 9.834 | 0.026 | 293E-21 | 223 | | | | | |
| 2 | 09.973 | 0.0890 | 461E-22 | 224 | | | | | |
| 3 | 10.104 | 0.0374 | 293E-20 | 225 | | | | | |

OUTPUT FOR SAMPLE CASE

CASE = .97 CQZ - .03 M2 (BY VOLUME), NO DEG. CONICAL BODY, HIGH VEL.

PRESTREAM PARAMETERS
 VELOCITY = 1.1175E+04 M/S
 DENSITY = 2.8500E-03 KG/M3
 PRESSURE = 1.1720E+02 N/M2
 ENTHALPY = 8.8000E+04 J/KG
 NOSE RADIUS = 5.0400E-01 M

BODY SHAPE, NONDIMENSIONAL BY NOSE RADIUS
 X/RND Y/RND Z/RND THETA R
 1 0.0000 0.0000 0.0000 1.5708 1.0000
 2 .2000 .0199 .1987 1.5708 1.0000
 3 .4000 .0769 .3494 1.5708 1.0000
 4 .6000 .1722 .5662 1.0472 0.0000
 5 .8000 .2722 .7394 1.0472 0.0000
 6 1.2000 .4722 1.0456 1.0472 0.0000
 7 2.0000 .8722 1.7706 1.0472 0.0000
 8 3.0000 1.3722 2.6446 1.0472 0.0000
 9 4.0000 1.8722 3.5107 1.0472 0.0000
 10 5.0000 2.3722 4.3767 1.0472 0.0000

PARAMETERS FOR NONDIMENSIONALIZATION
 VP = 1.1175E+04 M/S
 RNUF = 2.8500E-03 KG/M3
 VPMF = 1.2480E+04 J/KG
 RNDFAVPMF = 3.5591E+03 N/M2
 RNDFAVPMFVPMF = 3.0772E+05 N/M2
 VS = 1.1816E+04 K
 VPMF/VS = 1.0534E+04 J/KG-K
 RN = 3.0480E-01 M
 RV/VP = 2.7273E-05 S

NORMAL SHOCK PROPERTIES - NONDIMENSIONAL
 PRESSURE = .0375
 DENSITY = 15.93
 ENTHALPY = .6289
 TEMPERATURE = 1.000
 VELOCITY = -.0437
 MOLECULAR WT. = 13.29
 ENTROPY = 1.592
 SONIC VELOCITY = .2700

NAME2
 IFRAC = 0.8E+00,
 IASC = 500,
 CCL = 0.1E-03,
 CCF = 0.05E+00,
 ICE = 50,
 IASP = 1200,
 IOUTN = 200,
 IOUTF = 199,
 ITC = 100,
 IRC = 100,
 IKSU = 100, 200, 400, 600, 800, 1000,
 LTCU = 50, 50, 50, 50, 50,
 IRCU = 200, 200, 200, 200, 100,
 NCTS = 0,
 LSC = 809,
 CCS = 0.0E-03,
 GEND

CONVERGED OUTPUT ITERATIONS = 043 TIME = 1.519E+01

BODY STATION 1 S = 0.0000 THETA = 1.5708 DELTA = 0.0000 DELTA = .0456 DELTAT = 0.
 ARUD = 0.0000 RUDC = 0.0000 XSHUC = -.0456 YSHUC = .0000
 QUT = 0.910E-03 QUL = 4.739E-03 QML = 4.176E-03
 VELZ = 6.270E-02 SAZ = 0.

| RY | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| P | 0.179E-01 | 1.250E-01 | 2.500E-01 | 3.750E-01 | 5.000E-01 | 6.250E-01 | 7.500E-01 | 8.750E-01 | 1.000E+00 |
| RNU | 2.449E+01 | 1.700E+01 | 1.627E+01 | 1.779E+01 | 1.794E+01 | 1.714E+01 | 1.585E+01 | 1.452E+01 | 1.355E+01 |
| M | 3.100E-01 | 3.787E-01 | 3.912E-01 | 3.905E-01 | 4.054E-01 | 4.103E-01 | 4.144E-01 | 4.194E-01 | 4.249E-01 |
| U | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| V | 0. | -1.040E-02 | -1.803E-02 | -2.422E-02 | -3.010E-02 | -3.576E-02 | -4.125E-02 | -4.658E-02 | -5.170E-02 |
| N | 0. | 1.739E-02 | 6.030E-02 | 6.514E-02 | 9.190E-02 | 1.212E-01 | 1.530E-01 | 1.873E-01 | 2.232E-01 |
| T | 7.632E-01 | 9.109E-01 | 9.273E-01 | 9.537E-01 | 9.647E-01 | 9.733E-01 | 9.807E-01 | 9.866E-01 | 9.900E+00 |
| HN | 1.302E+01 | 1.400E+01 | 1.384E+01 | 1.373E+01 | 1.368E+01 | 1.360E+01 | 1.354E+01 | 1.348E+01 | 1.340E+01 |
| VA | 1.457E+00 | 1.540E+00 | 1.553E+00 | 1.562E+00 | 1.568E+00 | 1.574E+00 | 1.578E+00 | 1.584E+00 | 1.589E+00 |
| W | 2.201E-01 | 2.593E-01 | 2.580E-01 | 2.610E-01 | 2.635E-01 | 2.650E-01 | 2.664E-01 | 2.678E-01 | 2.700E-01 |
| Q | -2.544E-02 | -2.902E-02 | -2.926E-02 | -2.777E-02 | -2.602E-02 | -2.400E-02 | -2.162E-02 | -1.897E-02 | -1.590E-02 |
| UP | 0. | 1.767E-03 | 4.686E-03 | 5.390E-03 | 6.193E-03 | 6.511E-03 | 6.866E-03 | 7.142E-03 | 7.399E-03 |
| UN | 8.910E-03 | 9.51E-03 | 9.922E-03 | 9.596E-03 | 9.973E-03 | 9.004E-03 | 8.862E-03 | 5.029E-03 | 0. |
| QW | -8.910E-03 | -9.169E-03 | -9.286E-03 | -4.202E-03 | -1.978E-03 | 4.267E-04 | 3.107E-03 | 6.392E-03 | 1.299E-02 |
| QD | 1.178E-04 | 1.178E-04 | 3.155E-04 | 3.006E-04 | 6.024E-04 | 4.472E-04 | 5.497E-04 | 8.500E-04 | 1.674E+00 |
| RNUF | 4.016E-02 | 2.542E-03 | 2.594E-03 | 3.097E-03 | 1.597E-03 | 1.606E-03 | 1.108E-03 | 1.227E-03 | 0. |
| U | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |
| V | 0. | 4.279E-07 | 1.10E-06 | 7.080E-06 | 1.051E-05 | 1.219E-05 | 1.280E-05 | 2.257E-05 | 0. |
| N | 0. | 4.340E-05 | 3.780E-05 | 4.424E-05 | 6.024E-05 | 4.329E-05 | 4.320E-05 | 5.180E-05 | 0. |
| PI | -4.794E-04 | 6.104E-05 | -2.940E-05 | -9.408E-05 | -1.589E-05 | -1.977E-05 | -9.917E-06 | 3.189E-06 | 0. |
| TAU | 4.070E-02 | 1.623E-02 | 1.835E-02 | 1.883E-02 | 1.904E-02 | 1.956E-02 | 2.019E-02 | 2.093E-02 | 2.198E-02 |
| SE | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. | 0. |

MASS BALANCE PRESTREAM = 0. SHOCK LAYER = 0. DELTA MASS = 4.557E-02

BODY STATION 2 S = .2000 THETA = 1.3708 DELTA = .0219 DELTA = .0448 DELTAT = 0.
 XGCD = .019V YGCD = .1987 XSHUC = -.023V YSHUC = .2080
 QUT = 7.673E-03 QUL = 4.139E-03 QML = 3.554E-03
 VELZ = 1.906E-03 SAZ = 1.250E+00

| RY | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| P | 9.289E-01 | 1.250E-01 | 2.500E-01 | 3.750E-01 | 5.000E-01 | 6.250E-01 | 7.500E-01 | 8.750E-01 | 1.000E+00 |
| RNU | 2.349E+01 | 1.631E+01 | 1.786E+01 | 1.725E+01 | 1.699E+01 | 1.624E+01 | 1.569E+01 | 1.504E+01 | 1.435E+01 |
| M | 3.075E-01 | 3.752E-01 | 3.668E-01 | 3.942E-01 | 3.991E-01 | 4.023E-01 | 4.045E-01 | 4.064E-01 | 4.085E-01 |
| U | 0.627E-02 | 9.110E-02 | 1.082E-01 | 1.191E-01 | 1.322E-01 | 1.458E-01 | 1.590E-01 | 1.736E-01 | 1.877E-01 |
| V | 0. | -4.105E-03 | -4.729E-03 | -1.373E-02 | -2.646E-02 | -2.944E-02 | -3.407E-02 | -4.705E-02 | -5.558E-02 |
| N | 0. | 1.028E-01 | 9.014E-01 | 4.033E-01 | 5.140E-01 | 5.678E-01 | 6.245E-01 | 6.830E-01 | 7.440E-01 |
| T | 7.375E-01 | 9.014E-01 | 9.259E-01 | 9.110E-01 | 9.504E-01 | 9.565E-01 | 9.604E-01 | 9.636E-01 | 9.667E-01 |
| HN | 1.500E+01 | 1.404E+01 | 1.489E+01 | 1.360E+01 | 1.373E+01 | 1.349E+01 | 1.346E+01 | 1.340E+01 | 1.340E+01 |
| VA | 1.457E+00 | 1.540E+00 | 1.553E+00 | 1.562E+00 | 1.568E+00 | 1.574E+00 | 1.578E+00 | 1.584E+00 | 1.589E+00 |
| W | 2.180E-01 | 2.587E-01 | 2.589E-01 | 2.594E-01 | 2.611E-01 | 2.621E-01 | 2.628E-01 | 2.634E-01 | 2.643E-01 |
| Q | -2.706E-01 | -2.531E-01 | -2.717E-01 | -2.849E-01 | -2.939E-01 | -3.003E-01 | -3.053E-01 | -3.101E-01 | -3.190E-01 |
| UP | 0. | 1.576E-03 | 3.262E-03 | 4.746E-03 | 5.649E-03 | 6.564E-03 | 7.471E-03 | 8.364E-03 | 9.246E-03 |
| UN | 7.693E-03 | -0.949E-03 | -5.108E-03 | -3.351E-03 | -1.360E-03 | 7.588E-04 | 5.650E-03 | 5.650E-03 | 1.040E-02 |
| QW | 2.545E-02 | 2.603E-01 | 2.984E-01 | 3.223E-01 | 3.492E-01 | 3.778E-01 | 4.269E-01 | 4.547E-01 | 1.049E+00 |
| QD | 4.478E-02 | 4.164E-03 | 4.355E-03 | 4.859E-04 | 2.370E-03 | 1.944E-03 | 1.829E-04 | 2.122E-04 | 0. |
| U | 1.215E-05 | -8.057E-06 | -5.740E-05 | -5.364E-05 | -3.542E-05 | -1.092E-05 | 8.270E-06 | 1.829E-05 | 0. |
| V | 0. | -5.122E-06 | -3.398E-06 | 9.103E-06 | 2.167E-06 | 3.632E-06 | 4.489E-06 | 5.037E-06 | 0. |
| PI | 2.221E-06 | 2.709E-05 | 3.112E-05 | 2.607E-05 | 2.611E-05 | 2.623E-05 | 2.712E-05 | 2.884E-05 | 0. |
| HT | -3.714E-04 | -0.549E-06 | 0.388E-05 | 7.692E-05 | -1.502E-05 | -2.770E-05 | 6.292E-04 | 2.181E-05 | 0. |
| TAU | 2.139E-02 | 1.893E-02 | 1.895E-02 | 1.924E-02 | 1.967E-02 | 2.023E-02 | 2.095E-02 | 2.180E-02 | 2.299E-02 |
| SE | 0. | 0.017E-02 | 8.658E-02 | 1.341E-01 | 1.888E-01 | 2.484E-01 | 3.139E-01 | 3.830E-01 | 4.614E-01 |

MASS BALANCE PRESTREAM = 4.623E-01 SHOCK LAYER = 4.614E-01 DELTA MASS = 4.477E-02

BODY STATION 3 S = .4000 THETA = 1.1700 BETA = .0134 DELTA = .0505 DELTAF = 0.
 XBDU = .0789 YBDU = .3494 XSHOC = .0324 YSHOC = .4091
 QML = 4.713E-03 QML = 2.770E-03 QML = 1.943E-03
 VELZ = 3.800E-03 SAZ = 1.429E+00

| KV | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.250E-01 | 2.500E-01 | 3.750E-01 | 5.000E-01 | 6.250E-01 | 7.500E-01 | 8.750E-01 | 1.000E+00 |
| P | 7.837E-01 | 7.857E-01 | 7.876E-01 | 7.895E-01 | 7.914E-01 | 7.933E-01 | 7.952E-01 | 7.971E-01 | 7.990E-01 |
| PHO | 2.000E+01 | 1.625E+01 | 1.250E+01 | 8.75E+00 | 5.00E+00 | 1.25E+00 | 1.625E+00 | 2.000E+00 | 2.375E+00 |
| M | 2.986E-01 | 3.449E-01 | 3.912E-01 | 4.375E-01 | 4.838E-01 | 5.301E-01 | 5.764E-01 | 6.227E-01 | 6.690E-01 |
| U | 1.233E-01 | 1.671E-01 | 2.109E-01 | 2.547E-01 | 2.985E-01 | 3.423E-01 | 3.861E-01 | 4.299E-01 | 4.737E-01 |
| V | 0. | -4.035E-03 | -7.103E-03 | -1.190E-02 | -1.784E-02 | -2.404E-02 | -3.040E-02 | -3.691E-02 | -4.347E-02 |
| W | 5.255E-01 | 6.753E-01 | 8.251E-01 | 9.749E-01 | 1.124E+00 | 1.274E+00 | 1.424E+00 | 1.574E+00 | 1.724E+00 |
| T | 7.379E-01 | 8.713E-01 | 1.004E-01 | 1.131E-01 | 1.258E-01 | 1.385E-01 | 1.512E-01 | 1.639E-01 | 1.766E-01 |
| PH | 1.522E+01 | 1.413E+01 | 1.304E+01 | 1.195E+01 | 1.086E+01 | 9.771E+00 | 8.656E+00 | 7.541E+00 | 6.426E+00 |
| SA | 1.433E+00 | 1.536E+00 | 1.639E+00 | 1.742E+00 | 1.845E+00 | 1.948E+00 | 2.051E+00 | 2.154E+00 | 2.257E+00 |
| VA | 2.134E-01 | 2.476E-01 | 2.818E-01 | 3.160E-01 | 3.502E-01 | 3.844E-01 | 4.186E-01 | 4.528E-01 | 4.870E-01 |
| P1 | -3.374E-01 | -2.820E-01 | -2.266E-01 | -1.712E-01 | -1.158E-01 | -6.034E-02 | -4.713E-02 | -3.392E-02 | -2.071E-02 |
| QP | 0. | 1.061E-03 | 2.263E-03 | 3.465E-03 | 4.667E-03 | 5.869E-03 | 7.071E-03 | 8.273E-03 | 9.475E-03 |
| WM | 4.713E-03 | 5.157E-03 | 5.601E-03 | 6.045E-03 | 6.489E-03 | 6.933E-03 | 7.377E-03 | 7.821E-03 | 8.265E-03 |
| QK | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 |
| QD | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 |
| RMDT | 0. | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 |
| UT | 4.732E-04 | 9.464E-04 | 1.419E-03 | 2.128E-03 | 2.837E-03 | 3.546E-03 | 4.255E-03 | 4.964E-03 | 5.673E-03 |
| PT | 0.737E-05 | 1.474E-05 | 2.211E-05 | 2.948E-05 | 3.685E-05 | 4.422E-05 | 5.159E-05 | 5.896E-05 | 6.633E-05 |
| HT | -1.221E-02 | -1.157E-02 | -1.094E-02 | -1.031E-02 | -9.676E-03 | -9.041E-03 | -8.406E-03 | -7.771E-03 | -7.136E-03 |
| TAU | 2.346E-02 | 2.070E-02 | 1.794E-02 | 1.518E-02 | 1.242E-02 | 9.666E-03 | 6.890E-03 | 4.114E-03 | 1.338E-03 |
| SF | 0. | 1.283E-01 | 4.881E-01 | 4.247E-01 | 6.051E-01 | 1.151E-01 | 1.098E+00 | 1.373E+00 | 1.552E+00 |

MASS BALANCE PRESTREAM = 1.486E+00 SHOCK LAYER = 1.482E+00 DELTA MASS = 5.054E-02

BODY STATION 4 S = .4000 THETA = 1.0472 BETA = .0187 DELTA = .0525 DELTAF = 0.
 XBDU = .1722 YBDU = .3462 XSHOC = .1287 YSHOC = .5924
 QML = 2.174E-03 QML = 2.240E-03 QML = 1.304E-04
 VELZ = 4.861E-01 SAZ = 1.472E+00

| KV | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.250E-01 | 2.500E-01 | 3.750E-01 | 5.000E-01 | 6.250E-01 | 7.500E-01 | 8.750E-01 | 1.000E+00 |
| P | 7.433E-01 | 7.422E-01 | 7.411E-01 | 7.400E-01 | 7.389E-01 | 7.378E-01 | 7.367E-01 | 7.356E-01 | 7.345E-01 |
| PHO | 1.987E+01 | 1.566E+01 | 1.145E+01 | 7.23E+00 | 3.02E+00 | 1.75E+00 | 1.48E+00 | 1.21E+00 | 8.82E+00 |
| M | 2.932E-01 | 3.598E-01 | 4.270E-01 | 4.942E-01 | 5.614E-01 | 6.286E-01 | 6.958E-01 | 7.630E-01 | 8.302E-01 |
| U | 1.374E-01 | 1.653E-01 | 1.932E-01 | 2.211E-01 | 2.490E-01 | 2.769E-01 | 3.048E-01 | 3.327E-01 | 3.606E-01 |
| V | 0. | -2.211E-03 | -5.518E-03 | -8.825E-03 | -1.213E-02 | -1.544E-02 | -1.875E-02 | -2.206E-02 | -2.537E-02 |
| W | 6.508E-01 | 1.603E-01 | 1.228E-01 | 1.495E-01 | 1.717E+00 | 1.940E+00 | 2.163E+00 | 2.386E+00 | 2.609E+00 |
| T | 7.283E-01 | 8.562E-01 | 9.841E-01 | 1.112E-01 | 1.240E-01 | 1.368E-01 | 1.496E-01 | 1.624E-01 | 1.752E-01 |
| PH | 1.533E+01 | 1.622E+01 | 1.711E+01 | 1.800E+01 | 1.889E+01 | 1.978E+01 | 2.067E+01 | 2.156E+01 | 2.245E+01 |
| SA | 1.448E+00 | 1.533E+00 | 1.618E+00 | 1.703E+00 | 1.788E+00 | 1.873E+00 | 1.958E+00 | 2.043E+00 | 2.128E+00 |
| VA | 2.107E-01 | 2.449E-01 | 2.791E-01 | 3.133E-01 | 3.475E-01 | 3.817E-01 | 4.159E-01 | 4.501E-01 | 4.843E-01 |
| P1 | -3.477E-01 | -2.908E-01 | -2.339E-01 | -1.770E-01 | -1.201E-01 | -6.422E-02 | -4.713E-02 | -3.004E-02 | -1.295E-02 |
| QP | 0. | 1.195E-03 | 1.823E-03 | 2.451E-03 | 3.079E-03 | 3.707E-03 | 4.335E-03 | 4.963E-03 | 5.591E-03 |
| WM | 3.174E-03 | 3.425E-03 | 3.676E-03 | 3.927E-03 | 4.178E-03 | 4.429E-03 | 4.680E-03 | 4.931E-03 | 5.182E-03 |
| QK | -3.174E-03 | -3.174E-03 | -3.174E-03 | -3.174E-03 | -3.174E-03 | -3.174E-03 | -3.174E-03 | -3.174E-03 | -3.174E-03 |
| QD | 3.540E-02 | 3.540E-02 | 3.540E-02 | 3.540E-02 | 3.540E-02 | 3.540E-02 | 3.540E-02 | 3.540E-02 | 3.540E-02 |
| RMDT | -2.295E-02 | -3.078E-02 | -3.861E-02 | -4.644E-02 | -5.427E-02 | -6.210E-02 | -6.993E-02 | -7.776E-02 | -8.559E-02 |
| UT | 4.641E-04 | 9.282E-04 | 1.392E-03 | 2.138E-03 | 2.884E-03 | 3.630E-03 | 4.376E-03 | 5.122E-03 | 5.868E-03 |
| PT | 0. | 2.013E-05 | 4.026E-05 | 6.039E-05 | 8.052E-05 | 1.006E-04 | 1.207E-04 | 1.408E-04 | 1.609E-04 |
| HT | -8.643E-05 | -1.220E-05 | -1.678E-05 | -2.136E-05 | -2.594E-05 | -3.052E-05 | -3.510E-05 | -3.968E-05 | -4.426E-05 |
| SF | 3.386E-02 | 1.524E-02 | 2.439E-02 | 6.292E-02 | 1.004E-01 | 5.642E-02 | 1.352E-01 | 1.418E-01 | 0. |
| TAU | 2.492E-02 | 2.168E-02 | 1.844E-02 | 1.520E-02 | 1.196E-02 | 8.720E-03 | 5.484E-03 | 2.248E-03 | 0. |
| SF | 1.999E-01 | 1.999E-01 | 1.999E-01 | 1.999E-01 | 1.999E-01 | 1.999E-01 | 1.999E-01 | 1.999E-01 | 1.999E-01 |

MASS BALANCE PRESTREAM = 3.342E+00 SHOCK LAYER = 3.335E+00 DELTA MASS = 5.259E-02

BODY STATION 5 S = .4000 THETA = 1.0472 BETA = .0238 DELTA = .0580 DELTAF = 0.
 XBDU = .2722 YBDU = .7394 XSHOC = .2219 YSHOC = .7844
 QML = 2.682E-03 QML = 2.172E-03 QML = 7.097E-04
 VELZ = 4.749E-01 SAZ = 1.448E+00

| KV | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.250E-01 | 2.500E-01 | 3.750E-01 | 5.000E-01 | 6.250E-01 | 7.500E-01 | 8.750E-01 | 1.000E+00 |
| P | 7.433E-01 | 7.422E-01 | 7.411E-01 | 7.400E-01 | 7.389E-01 | 7.378E-01 | 7.367E-01 | 7.356E-01 | 7.345E-01 |
| PHO | 2.035E+01 | 1.604E+01 | 1.173E+01 | 7.42E+00 | 3.11E+00 | 1.84E+00 | 1.57E+00 | 1.30E+00 | 8.72E+00 |
| M | 2.808E-01 | 3.474E-01 | 4.140E-01 | 4.806E-01 | 5.472E-01 | 6.138E-01 | 6.804E-01 | 7.470E-01 | 8.136E-01 |
| U | 1.252E-01 | 1.531E-01 | 1.810E-01 | 2.089E-01 | 2.368E-01 | 2.647E-01 | 2.926E-01 | 3.205E-01 | 3.484E-01 |
| V | 0. | -4.089E-03 | -7.178E-03 | -1.102E-02 | -1.486E-02 | -1.870E-02 | -2.254E-02 | -2.638E-02 | -3.022E-02 |
| W | 6.479E-01 | 1.013E-01 | 1.284E+00 | 1.429E+00 | 1.474E+00 | 2.029E+00 | 2.110E+00 | 2.177E+00 | 2.244E+00 |
| T | 7.421E-01 | 8.755E-01 | 1.008E-01 | 1.141E-01 | 1.274E-01 | 1.407E-01 | 1.540E-01 | 1.673E-01 | 1.806E-01 |
| PH | 1.544E+01 | 1.633E+01 | 1.722E+01 | 1.811E+01 | 1.900E+01 | 1.989E+01 | 2.078E+01 | 2.167E+01 | 2.256E+01 |
| SA | 1.441E+00 | 1.526E+00 | 1.611E+00 | 1.696E+00 | 1.781E+00 | 1.866E+00 | 1.951E+00 | 2.036E+00 | 2.121E+00 |
| VA | 2.086E-01 | 2.428E-01 | 2.770E-01 | 3.112E-01 | 3.454E-01 | 3.796E-01 | 4.138E-01 | 4.480E-01 | 4.822E-01 |
| P1 | -3.374E-01 | -2.820E-01 | -2.266E-01 | -1.712E-01 | -1.158E-01 | -6.034E-02 | -4.713E-02 | -3.392E-02 | -2.071E-02 |
| QP | 0. | 1.061E-03 | 2.263E-03 | 3.465E-03 | 4.667E-03 | 5.869E-03 | 7.071E-03 | 8.273E-03 | 9.475E-03 |
| WM | 4.713E-03 | 5.157E-03 | 5.601E-03 | 6.045E-03 | 6.489E-03 | 6.933E-03 | 7.377E-03 | 7.821E-03 | 8.265E-03 |
| QK | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 | 4.713E-03 |
| QD | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 | 3.418E-02 |
| RMDT | 0. | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 | 0.457E-03 |
| UT | 4.732E-04 | 9.464E-04 | 1.419E-03 | 2.128E-03 | 2.837E-03 | 3.546E-03 | 4.255E-03 | 4.964E-03 | 5.673E-03 |
| PT | 0.737E-05 | 1.474E-05 | 2.211E-05 | 2.948E-05 | 3.685E-05 | 4.422E-05 | 5.159E-05 | 5.896E-05 | 6.633E-05 |
| HT | -1.221E-02 | -1.157E-02 | -1.094E-02 | -1.031E-02 | -9.676E-03 | -9.041E-03 | -8.406E-03 | -7.771E-03 | -7.136E-03 |
| TAU | 2.346E-02 | 2.070E-02 | 1.794E-02 | 1.518E-02 | 1.242E-02 | 9.666E-03 | 6.890E-03 | 4.114E-03 | 1.338E-03 |
| SF | 0. | 1.283E-01 | 4.881E-01 | 4.247E-01 | 6.051E-01 | 1.151E-01 | 1.098E+00 | 1.373E+00 | 1.552E+00 |

MASS BALANCE PRESTREAM = 5.088E+00 SHOCK LAYER = 5.080E+00 DELTA MASS = 5.802E-02

BODY STATION 6 S = 1.2000 THETA = 1.0472 BETA = .0484 DELTA = .0765 DELTAF = 0.
 XBDU = .6722 YBDU = 1.0858 XSHOC = .4059 YSHOC = 1.1241
 QML = 2.803E-03 QML = 2.247E-03 QML = 5.761E-04
 VELZ = 4.801E-01 SAZ = 1.466E+00

| KV | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|-----------|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.250E-01 | 2.500E-01 | 3.750E-01 | 5.000E-01 | 6.250E-01 | 7.500E-01 | 8.750E-01 | 1.000E+00 |
| P | 7.433E-01 | 7.422E-01 | 7.411E-01 | 7.400E-01 | 7.389E-01 | 7.378E-01 | 7.367E-01 | 7.356E-01 | 7.345E-01 |
| PHO | 2.131E+01 | 1.701E+01 | 1.271E+01 | 8.40E+00 | 4.09E+00 | 2.78E+00 | 1.47E+00 | 1.16E+00 | 8.26E+00 |
| M | 2.768E-01 | 3.434E-01 | 4.100E-01 | 4.766E-01 | 5.432E-01 | 6.098E-01 | 6.764E-01 | 7.430E-01 | 8.096E-01 |
| U | 1.321E-01 | 1.600E-01 | 1.879E-01 | 2.158E-01 | 2.437E-01 | 2.716E-01 | 2.995E-01 | 3.274E-01 | 3.553E-01 |
| V | 0. | -2.211E-03 | -5.518E-03 | -8.825E-03 | -1.213E-02 | -1.544E-02 | -1.875E-02 | -2.206E-02 | -2.537E-02 |
| W | 6.508E-01 | 1.603E-01 | 1.228E-01 | 1.495E-01 | 1.717E+00 | 1.940E+00 | 2.163E+00 | 2.386E+ | |

BOUY STATION 7 S = 4.0000 INETA = 1.0472 BETA = .0594 DELTA = -1.186 DELTAT = 0.
 ABUD = .8722 YBUU = 1.7786 XSMOC = .7803 YSMOC = 1.8280
 QUT = 3.094E-03 QUC = 2.533E-03 QUL = 5.612E-04
 VEL2 = 4.558E-01 SAZ = 1.462E+00

| BY | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|----|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.450E-01 | 2.540E-01 | 3.730E-01 | 5.000E-01 | 6.430E-01 | 7.500E-01 | 8.730E-01 | 1.000E+00 |
| P | 0. | 7.643E-01 | 7.577E-01 | 7.567E-01 | 7.548E-01 | 7.529E-01 | 7.477E-01 | 7.309E-01 | 7.000E-01 |
| AMU | 0. | 2.254E+01 | 1.815E+01 | 1.457E+01 | 1.199E+01 | 9.436E+00 | 6.872E+00 | 4.308E+00 | 1.803E+00 |
| M | 0. | 4.897E-01 | 3.453E-01 | 3.193E-01 | 3.186E-01 | 3.193E-01 | 3.208E-01 | 3.206E-01 | 3.203E-01 |
| U | 0. | 1.104E-01 | 3.865E-01 | 4.545E-01 | 4.554E-01 | 4.573E-01 | 4.588E-01 | 4.595E-01 | 4.595E-01 |
| V | 0. | -1.046E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 |
| W | 0. | 6.585E-01 | 1.705E+00 | 2.014E+00 | 2.041E+00 | 2.045E+00 | 2.048E+00 | 2.051E+00 | 2.051E+00 |
| X | 0. | 9.950E-01 | 7.764E-01 | 7.686E-01 | 7.674E-01 | 7.670E-01 | 7.712E-01 | 7.721E-01 | 7.721E-01 |
| Y | 0. | 1.618E-01 | 1.473E+01 | 1.482E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 |
| ZA | 0. | 1.399E+00 | 1.492E+00 | 1.482E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 |
| VB | 0. | 1.985E-01 | 2.208E-01 | 2.237E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 |
| PI | 0. | -4.006E-01 | -2.948E-01 | -3.038E-01 | -3.058E-01 | -3.063E-01 | -3.063E-01 | -3.063E-01 | -3.063E-01 |
| QD | 0. | 1.154E-01 | 3.752E-01 | 4.142E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 |
| UH | 0. | 3.094E-03 | 3.215E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 |
| QM | 0. | 2.994E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 |
| UD | 0. | 6.230E-01 | 6.027E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 |
| RMU3 | 0. | 1.079E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 |
| UI | 0. | 1.039E-04 | 5.942E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 |
| VE | 0. | 1.980E-05 | 7.344E-06 | 3.138E-06 | 6.441E-07 | 7.722E-07 | 1.023E-06 | 1.474E-06 | 1.880E-06 |
| PT | 0. | -1.567E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 |
| HT | 0. | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 |
| TAU | 0. | 1.110E+00 | 4.880E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 |
| SP | 0. | 1.110E+00 | 4.880E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 |

MASS BALANCE PRESTREAM = 1.422E+01 SHOCK LAYER = 1.420E+01 DELTA MASS = 1.488E-01

BOUY STATION 8 S = 3.0000 INETA = 1.0472 BETA = .0594 DELTA = -1.186 DELTAT = 0.
 ABUD = 1.7722 YBUU = 2.4444 XSMOC = 1.1223 YSMOC = 1.7341
 QUT = 3.024E-03 QUC = 3.040E-03 QUL = 5.623E-04
 VEL2 = 4.550E-01 SAZ = 1.462E+00

| BY | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|----|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.450E-01 | 2.540E-01 | 3.730E-01 | 5.000E-01 | 6.430E-01 | 7.500E-01 | 8.730E-01 | 1.000E+00 |
| P | 0. | 7.643E-01 | 7.577E-01 | 7.567E-01 | 7.548E-01 | 7.529E-01 | 7.477E-01 | 7.309E-01 | 7.000E-01 |
| AMU | 0. | 2.254E+01 | 1.815E+01 | 1.457E+01 | 1.199E+01 | 9.436E+00 | 6.872E+00 | 4.308E+00 | 1.803E+00 |
| M | 0. | 4.897E-01 | 3.453E-01 | 3.193E-01 | 3.186E-01 | 3.193E-01 | 3.208E-01 | 3.206E-01 | 3.203E-01 |
| U | 0. | 1.104E-01 | 3.865E-01 | 4.545E-01 | 4.554E-01 | 4.573E-01 | 4.588E-01 | 4.595E-01 | 4.595E-01 |
| V | 0. | -1.046E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 |
| W | 0. | 6.585E-01 | 1.705E+00 | 2.014E+00 | 2.041E+00 | 2.045E+00 | 2.048E+00 | 2.051E+00 | 2.051E+00 |
| X | 0. | 9.950E-01 | 7.764E-01 | 7.686E-01 | 7.674E-01 | 7.670E-01 | 7.712E-01 | 7.721E-01 | 7.721E-01 |
| Y | 0. | 1.618E-01 | 1.473E+01 | 1.482E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 |
| ZA | 0. | 1.399E+00 | 1.492E+00 | 1.482E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 |
| VB | 0. | 1.985E-01 | 2.208E-01 | 2.237E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 |
| PI | 0. | -4.006E-01 | -2.948E-01 | -3.038E-01 | -3.058E-01 | -3.063E-01 | -3.063E-01 | -3.063E-01 | -3.063E-01 |
| QD | 0. | 1.154E-01 | 3.752E-01 | 4.142E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 |
| UH | 0. | 3.094E-03 | 3.215E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 |
| QM | 0. | 2.994E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 |
| UD | 0. | 6.230E-01 | 6.027E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 |
| RMU3 | 0. | 1.079E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 |
| UI | 0. | 1.039E-04 | 5.942E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 |
| VE | 0. | 1.980E-05 | 7.344E-06 | 3.138E-06 | 6.441E-07 | 7.722E-07 | 1.023E-06 | 1.474E-06 | 1.880E-06 |
| PT | 0. | -1.567E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 |
| HT | 0. | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 |
| TAU | 0. | 1.110E+00 | 4.880E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 |
| SP | 0. | 1.110E+00 | 4.880E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 |

MASS BALANCE PRESTREAM = 1.419E+01 SHOCK LAYER = 1.418E+01 DELTA MASS = 1.480E-01

BOUY STATION 9 S = 4.0000 INETA = 1.0472 BETA = .0594 DELTA = -1.186 DELTAT = 0.
 ABUD = 1.8722 YBUU = 3.3107 XSMOC = 1.6752 YSMOC = 3.6244
 QUT = 4.040E-03 QUC = 3.536E-03 QUL = 6.075E-04
 VEL2 = 4.550E-01 SAZ = 1.462E+00

| BY | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|------|----|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.450E-01 | 2.540E-01 | 3.730E-01 | 5.000E-01 | 6.430E-01 | 7.500E-01 | 8.730E-01 | 1.000E+00 |
| P | 0. | 7.643E-01 | 7.577E-01 | 7.567E-01 | 7.548E-01 | 7.529E-01 | 7.477E-01 | 7.309E-01 | 7.000E-01 |
| AMU | 0. | 2.254E+01 | 1.815E+01 | 1.457E+01 | 1.199E+01 | 9.436E+00 | 6.872E+00 | 4.308E+00 | 1.803E+00 |
| M | 0. | 4.897E-01 | 3.453E-01 | 3.193E-01 | 3.186E-01 | 3.193E-01 | 3.208E-01 | 3.206E-01 | 3.203E-01 |
| U | 0. | 1.104E-01 | 3.865E-01 | 4.545E-01 | 4.554E-01 | 4.573E-01 | 4.588E-01 | 4.595E-01 | 4.595E-01 |
| V | 0. | -1.046E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 |
| W | 0. | 6.585E-01 | 1.705E+00 | 2.014E+00 | 2.041E+00 | 2.045E+00 | 2.048E+00 | 2.051E+00 | 2.051E+00 |
| X | 0. | 9.950E-01 | 7.764E-01 | 7.686E-01 | 7.674E-01 | 7.670E-01 | 7.712E-01 | 7.721E-01 | 7.721E-01 |
| Y | 0. | 1.618E-01 | 1.473E+01 | 1.482E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 | 1.484E+01 |
| ZA | 0. | 1.399E+00 | 1.492E+00 | 1.482E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 | 1.481E+00 |
| VB | 0. | 1.985E-01 | 2.208E-01 | 2.237E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 | 2.232E-01 |
| PI | 0. | -4.006E-01 | -2.948E-01 | -3.038E-01 | -3.058E-01 | -3.063E-01 | -3.063E-01 | -3.063E-01 | -3.063E-01 |
| QD | 0. | 1.154E-01 | 3.752E-01 | 4.142E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 | 4.170E-01 |
| UH | 0. | 3.094E-03 | 3.215E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 | 3.212E-03 |
| QM | 0. | 2.994E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 | 2.210E-03 |
| UD | 0. | 6.230E-01 | 6.027E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 | 6.010E-01 |
| RMU3 | 0. | 1.079E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 | 1.234E-02 |
| UI | 0. | 1.039E-04 | 5.942E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 | 1.548E-04 |
| VE | 0. | 1.980E-05 | 7.344E-06 | 3.138E-06 | 6.441E-07 | 7.722E-07 | 1.023E-06 | 1.474E-06 | 1.880E-06 |
| PT | 0. | -1.567E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 | -1.043E-05 |
| HT | 0. | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 | 3.949E-02 |
| TAU | 0. | 1.110E+00 | 4.880E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 |
| SP | 0. | 1.110E+00 | 4.880E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 | 4.742E+00 |

MASS BALANCE PRESTREAM = 1.447E+01 SHOCK LAYER = 1.446E+01 DELTA MASS = 2.275E-01

BOUY STATION 10 S = 5.0000 INETA = 1.0472 BETA = .0594 DELTA = -1.186 DELTAT = 0.
 ABUD = 2.8722 YBUU = 4.3707 XSMOC = 2.1283 YSMOC = 4.5175
 QUT = 4.040E-03 QUC = 4.024E-03 QUL = 6.369E-04
 VEL2 = 4.550E-01 SAZ = 1.462E+00

| BY | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-----|----|------------|------------|------------|------------|------------|------------|------------|------------|
| T | 0. | 1.450E-01 | 2.540E-01 | 3.730E-01 | 5.000E-01 | 6.430E-01 | 7.500E-01 | 8.730E-01 | 1.000E+00 |
| P | 0. | 7.643E-01 | 7.577E-01 | 7.567E-01 | 7.548E-01 | 7.529E-01 | 7.477E-01 | 7.309E-01 | 7.000E-01 |
| AMU | 0. | 2.254E+01 | 1.815E+01 | 1.457E+01 | 1.199E+01 | 9.436E+00 | 6.872E+00 | 4.308E+00 | 1.803E+00 |
| M | 0. | 4.897E-01 | 3.453E-01 | 3.193E-01 | 3.186E-01 | 3.193E-01 | 3.208E-01 | 3.206E-01 | 3.203E-01 |
| U | 0. | 1.104E-01 | 3.865E-01 | 4.545E-01 | 4.554E-01 | 4.573E-01 | 4.588E-01 | 4.595E-01 | 4.595E-01 |
| V | 0. | -1.046E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 | -6.040E-01 |
| W | 0. | 6.585E-01 | 1.705E+00 | 2.014E+00 | 2.041E+00 | 2.045E+00 | 2.048E+00 | 2.051E+00 | 2.051E+00 |
| X | 0. | 9.950E-01 | 7.764E-01 | 7.686E-01 | 7.674E-01 | 7.670E- | | | |